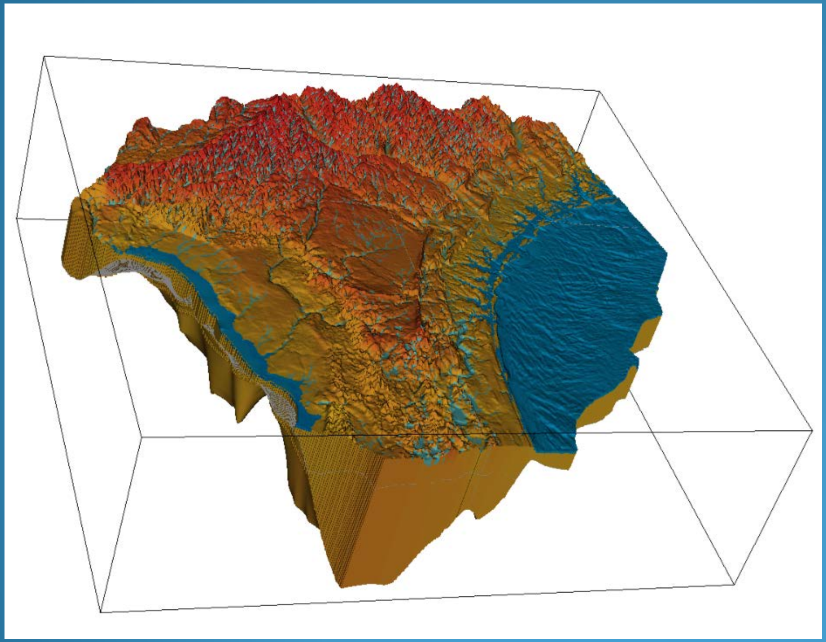
NFSEG Version 1.1 Uncertainty Analysis



April 18, 2018

Outline

- Why do uncertainty analyses?
- How do we go about estimating uncertainty with the NFSEG model?
  - Theoretical basis
  - Methods of analysis
- What were our results?

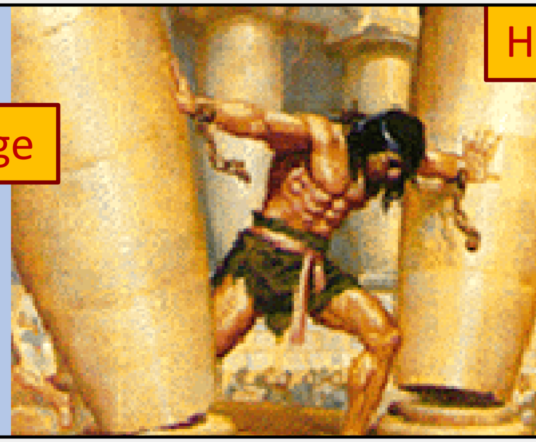
Background and Motivation

The fundamental question that underpins all decision-making

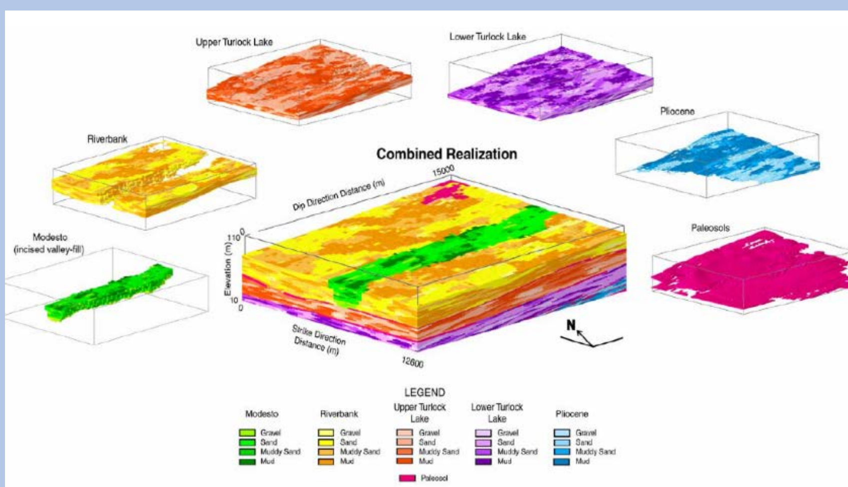
Model parameterization
the two pillars on which it rests

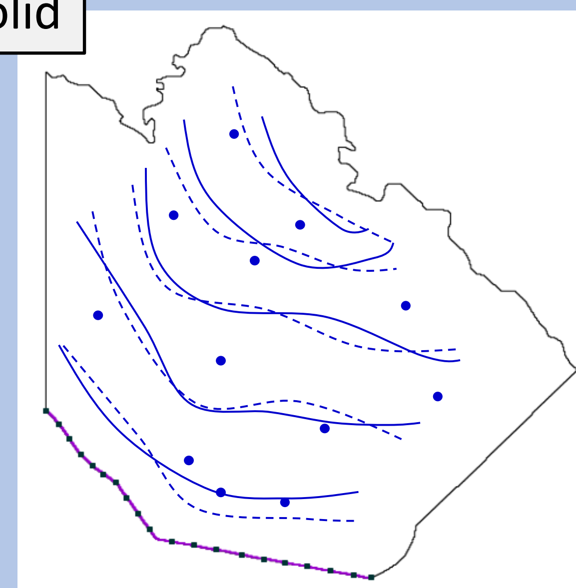
History-matching

Expert knowledge



Neither of these is very solid





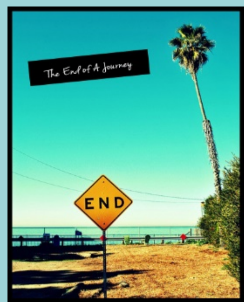
Expert knowledge is a stochastic quantity

The greater the detail that we express the less we know the exact value

There are infinite ways to fit a calibration dataset

Each can lead to different predictions

Bayes Equation



History-matching

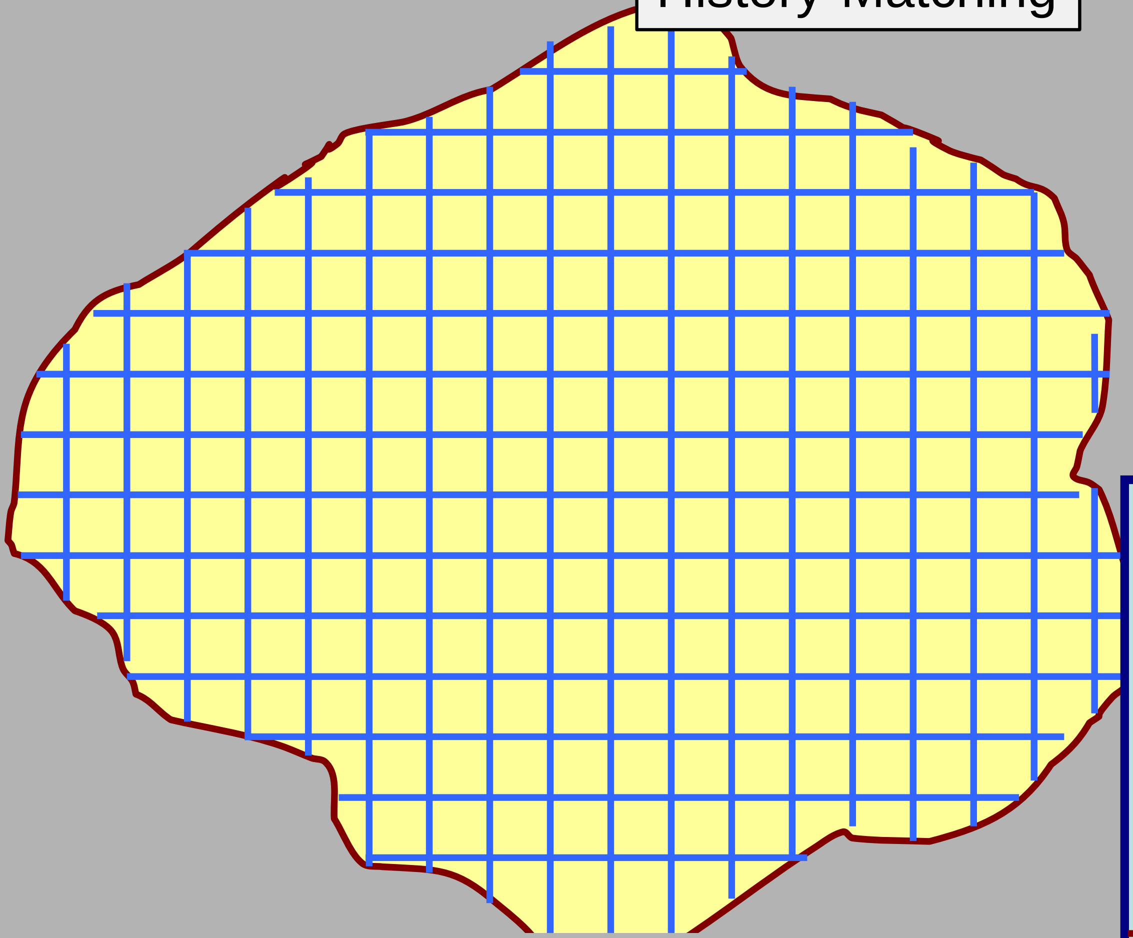
$$P(k|h) \propto P(h|k)\, P(k)$$

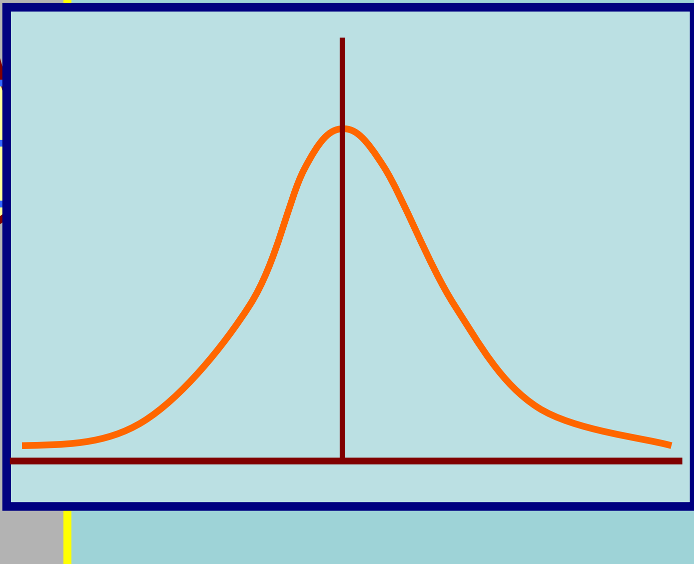
The possibilities that remain

Which of these possibilities fits the data

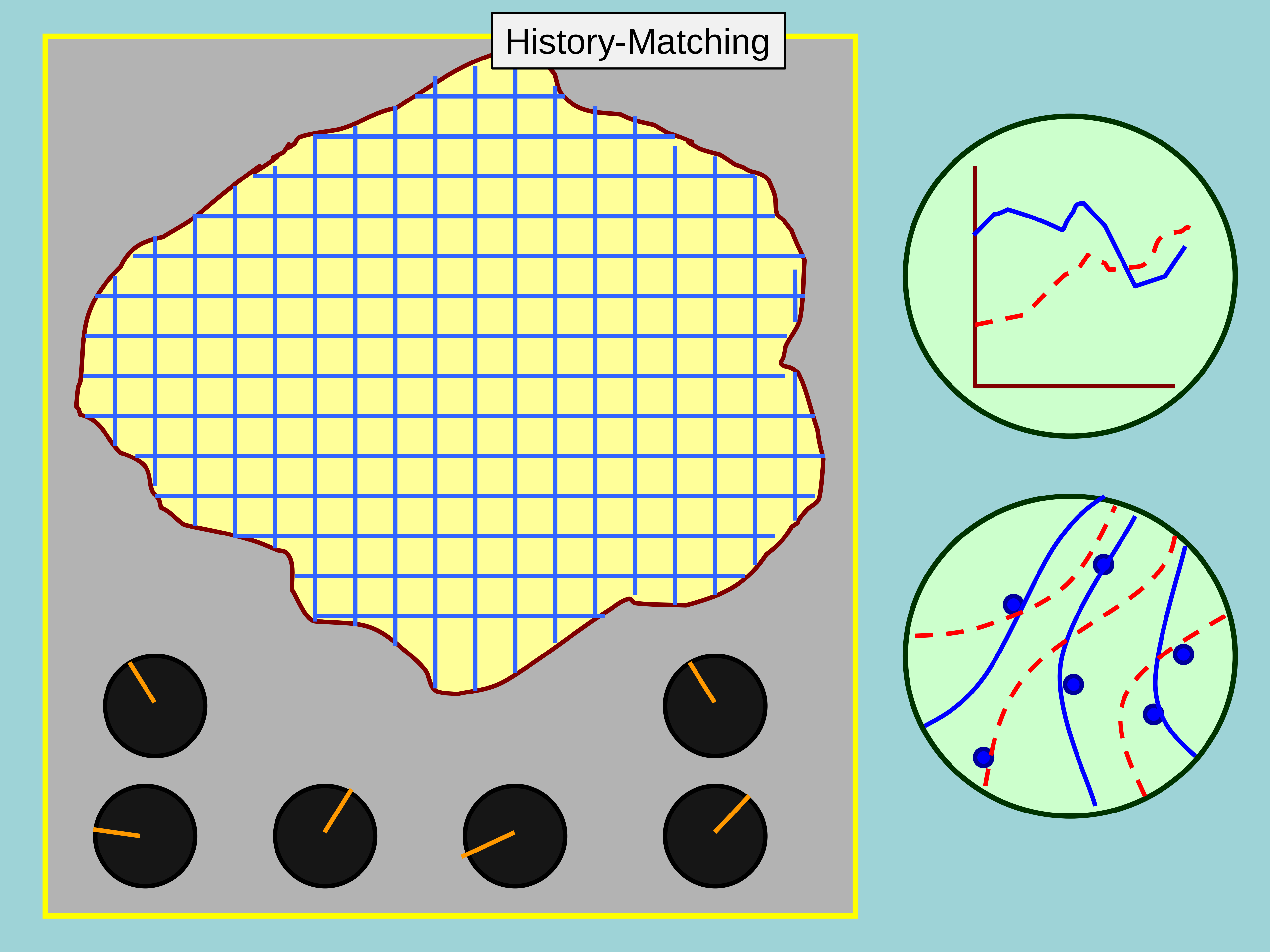
What is possible based on expert knowledge

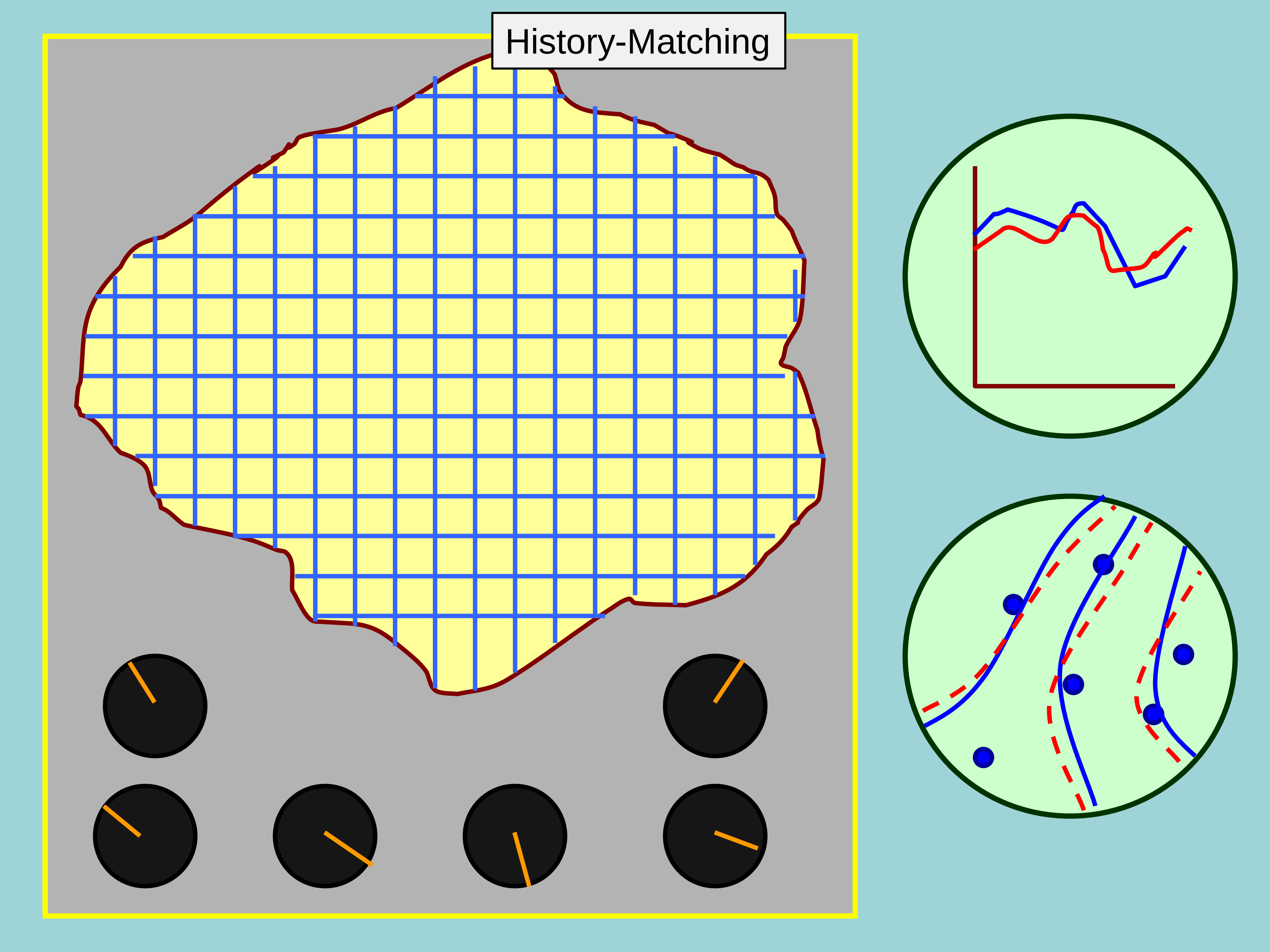
History-Matching



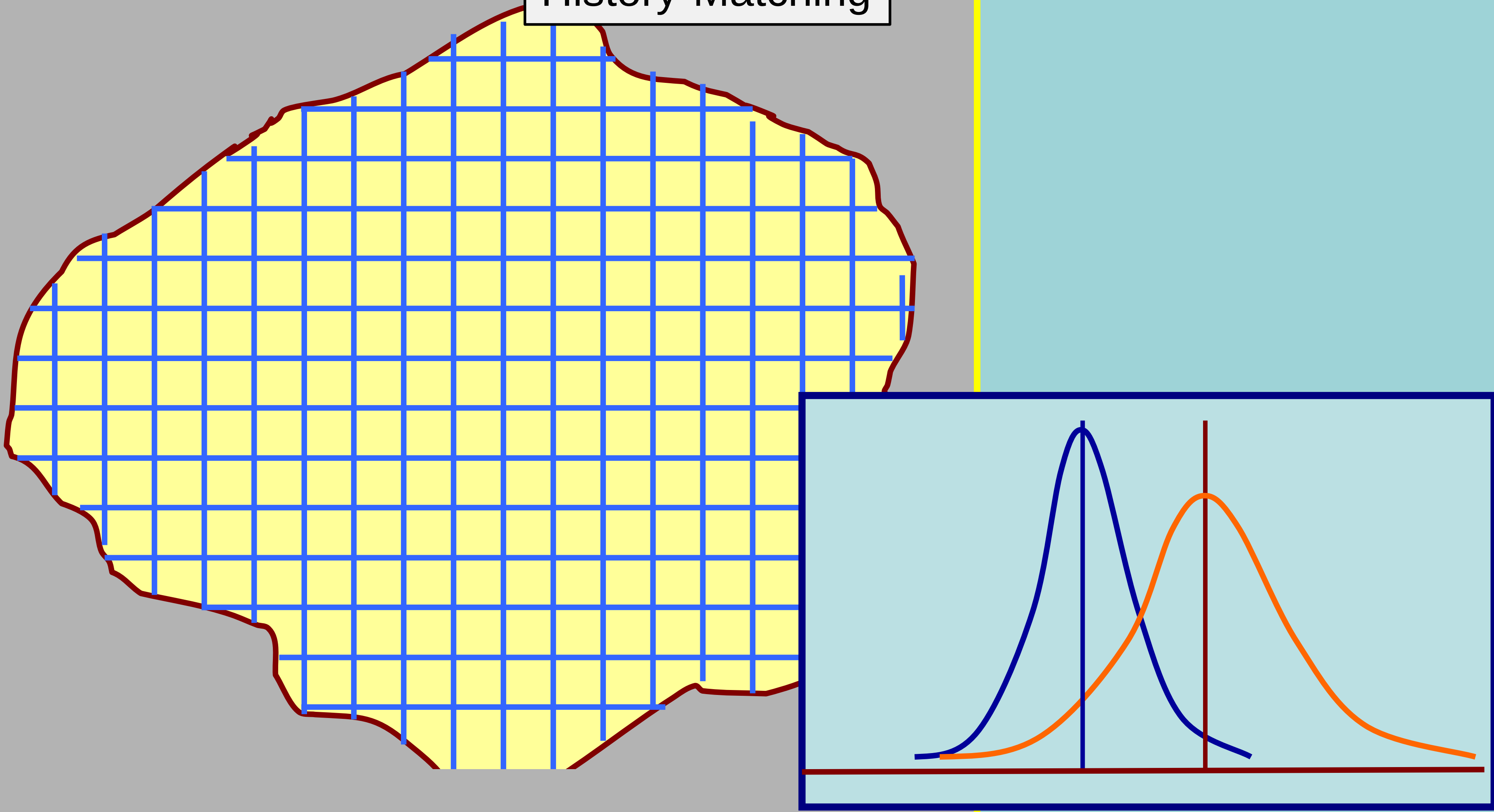


Before


History-Matching


History-Matching

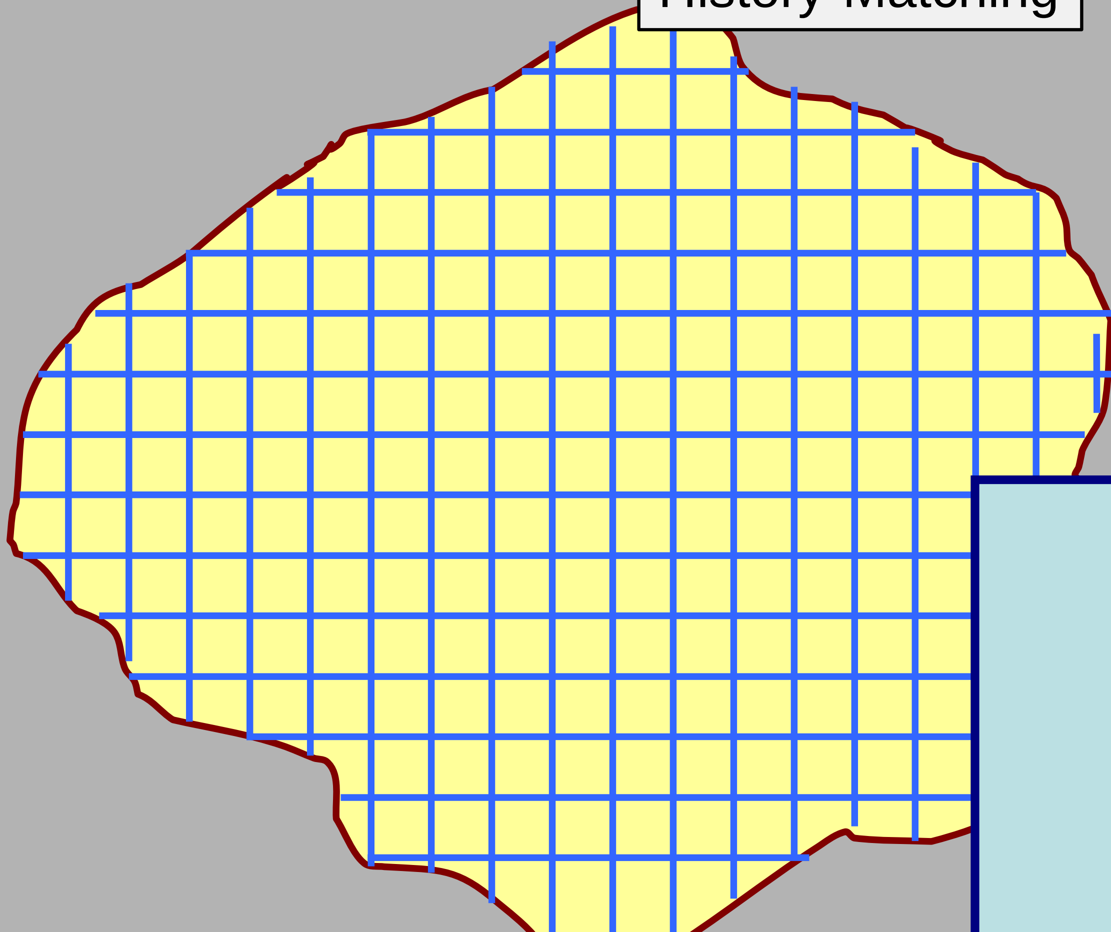
History-Matching

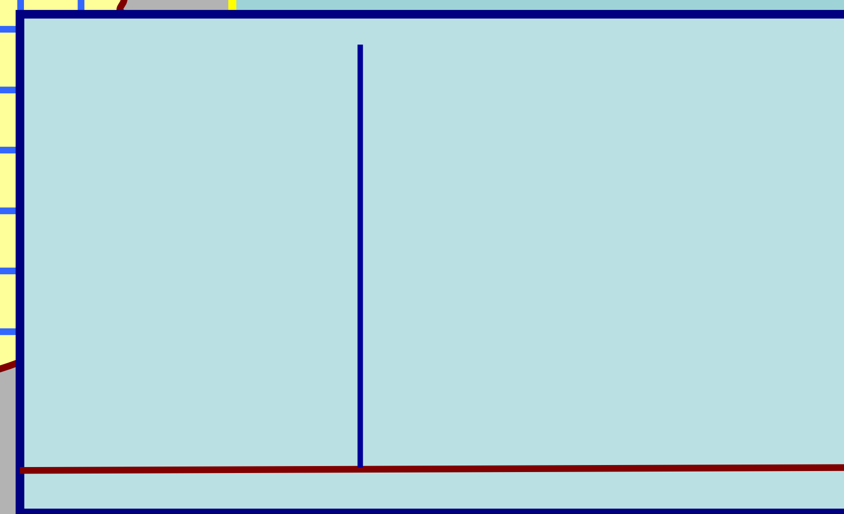


After

So what does the "calibrated model" give us?

History-Matching

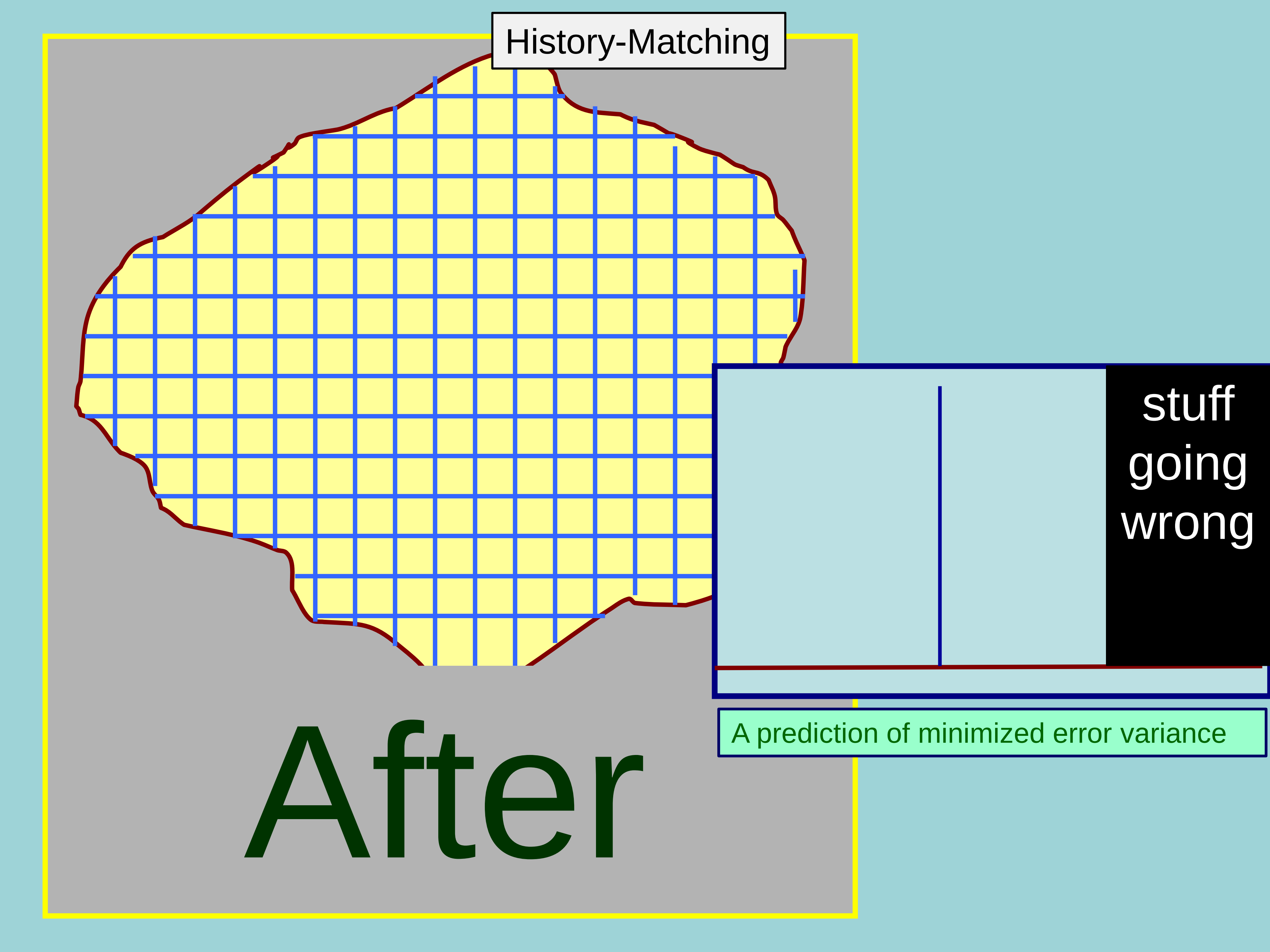




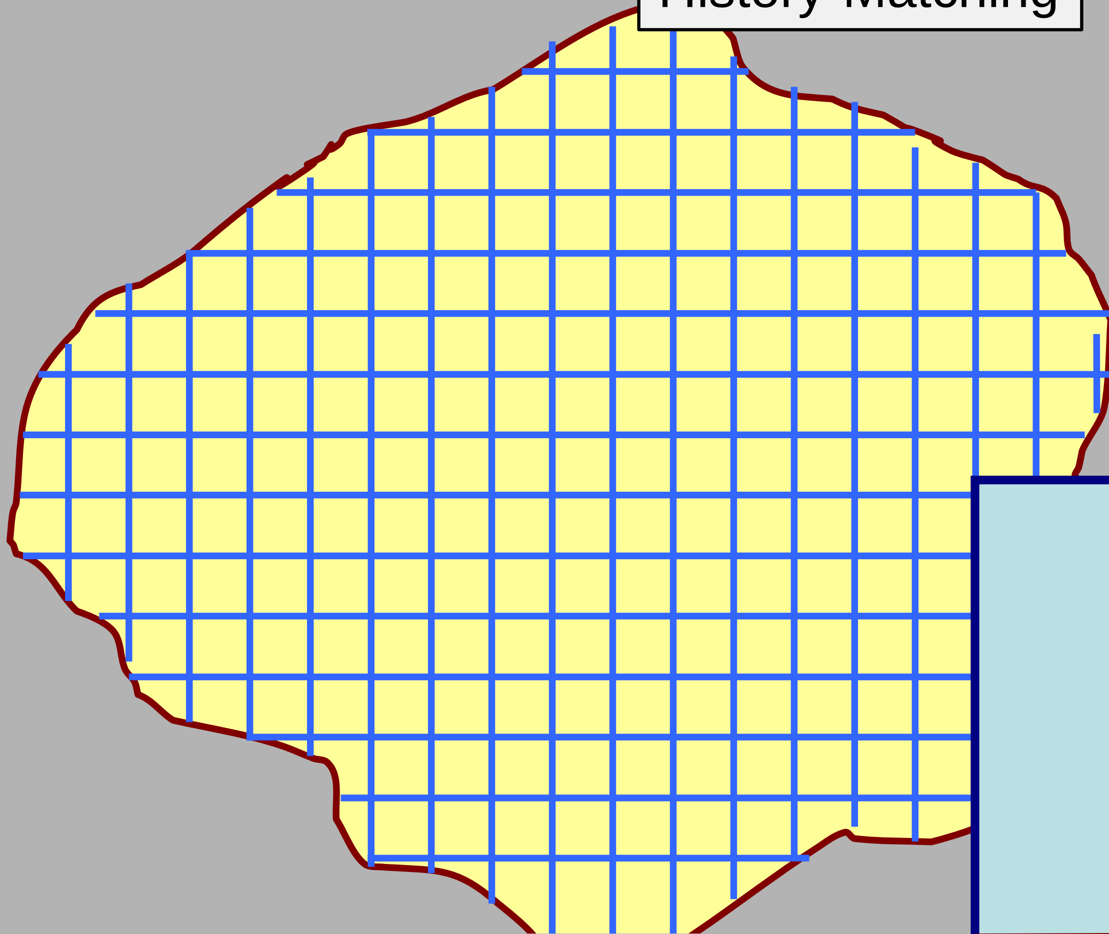
A prediction of minimized error variance

After

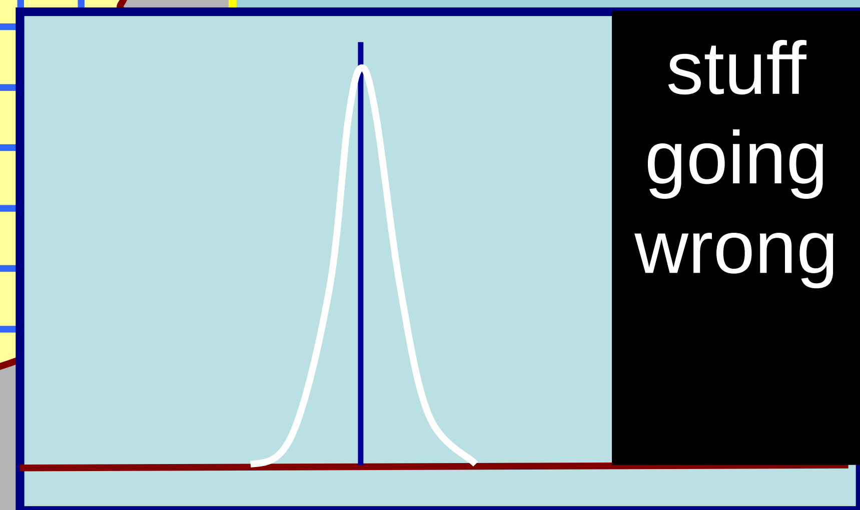

What can go wrong?


History-Matching
stuff going wrong
A prediction of minimized error variance
After

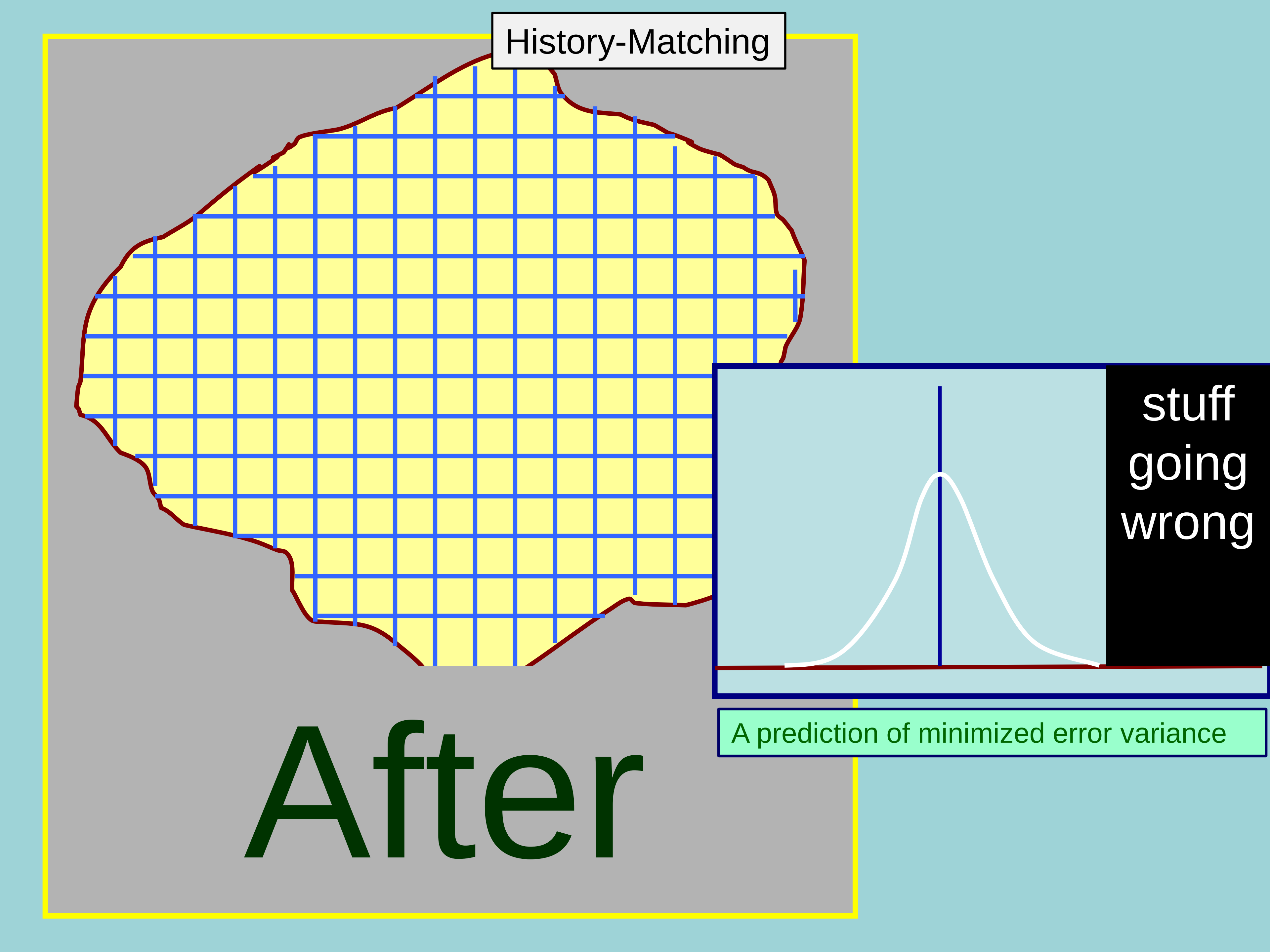
History-Matching

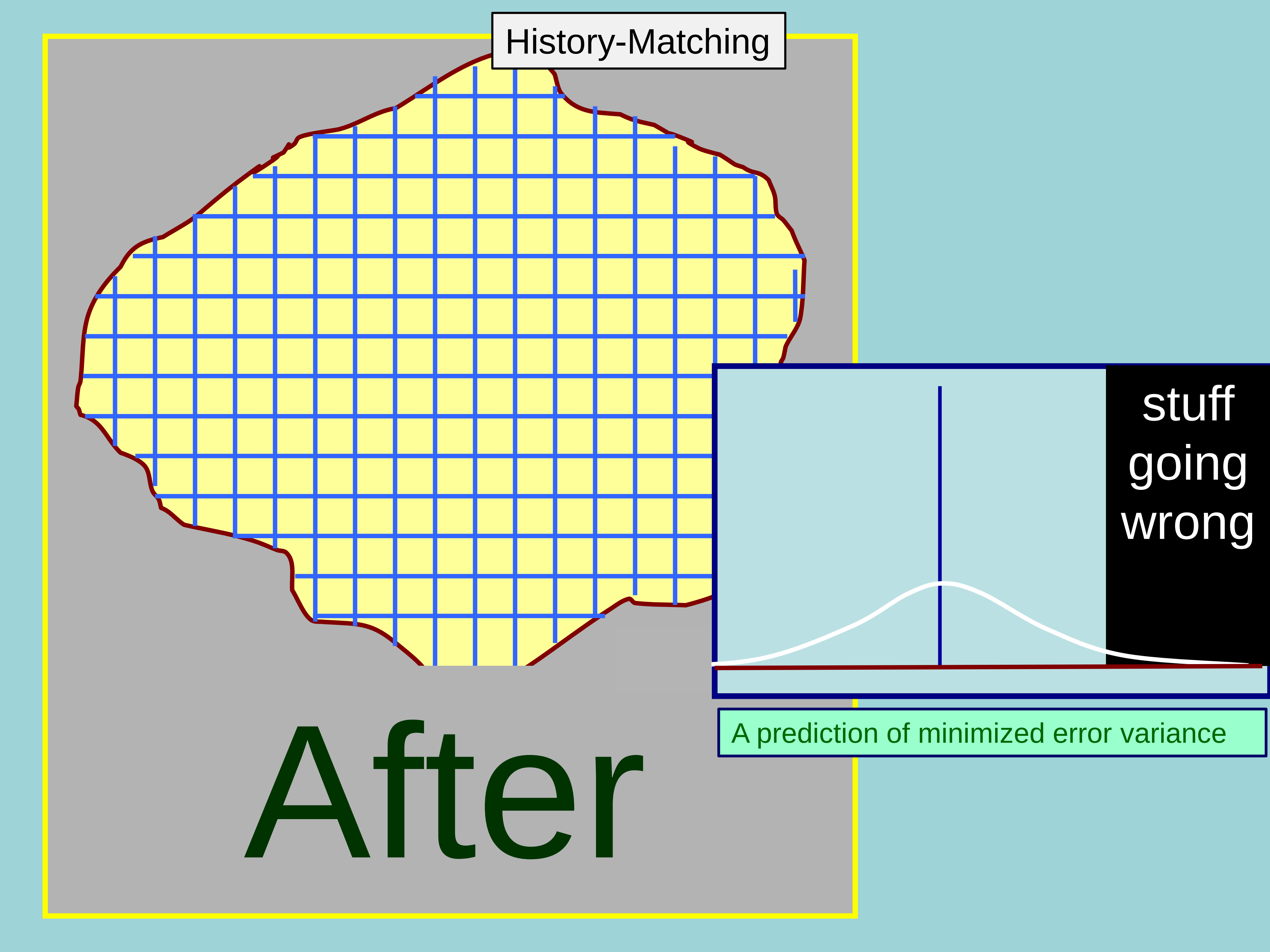


After



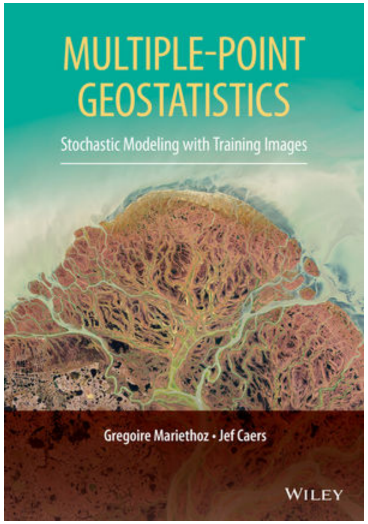
A prediction of minimized error variance

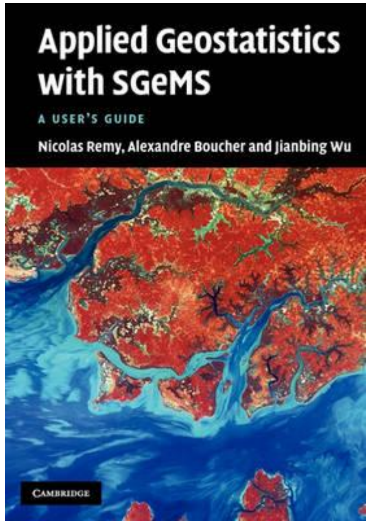

History-Matching
stuff going wrong
A prediction of minimized error variance
After

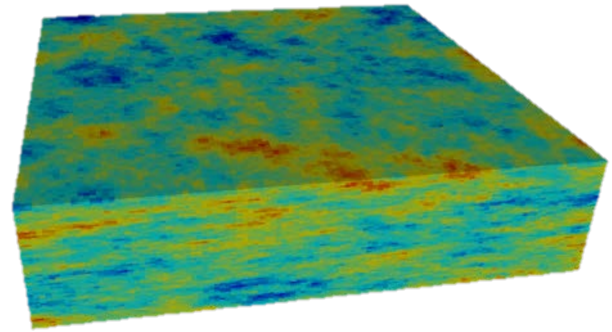

History-Matching
stuff going wrong
A prediction of minimized error variance
After

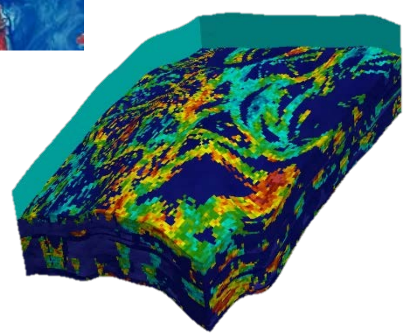
Prior Probability Distribution

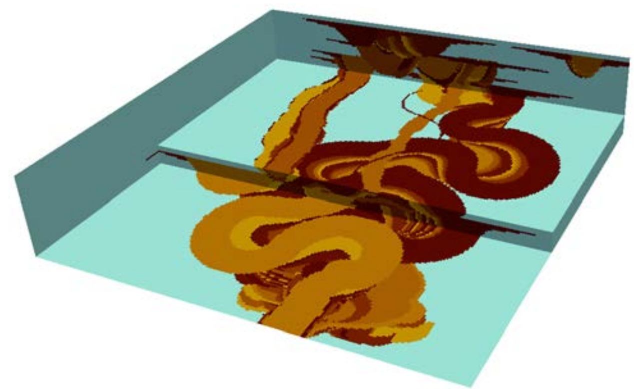
Geostatistics: generating realisations of prior probability distributions

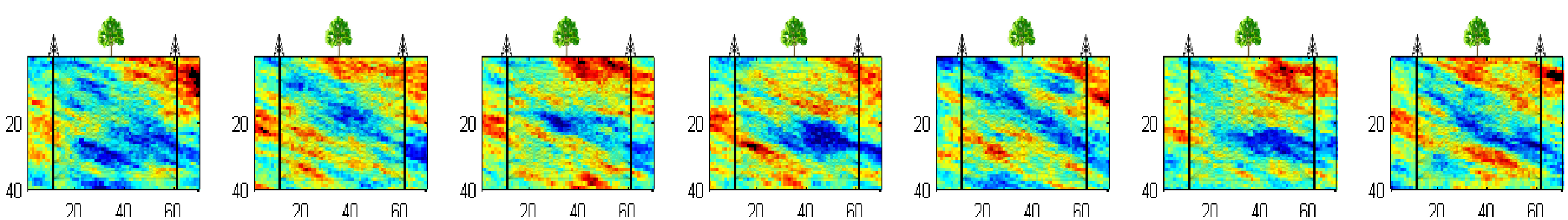






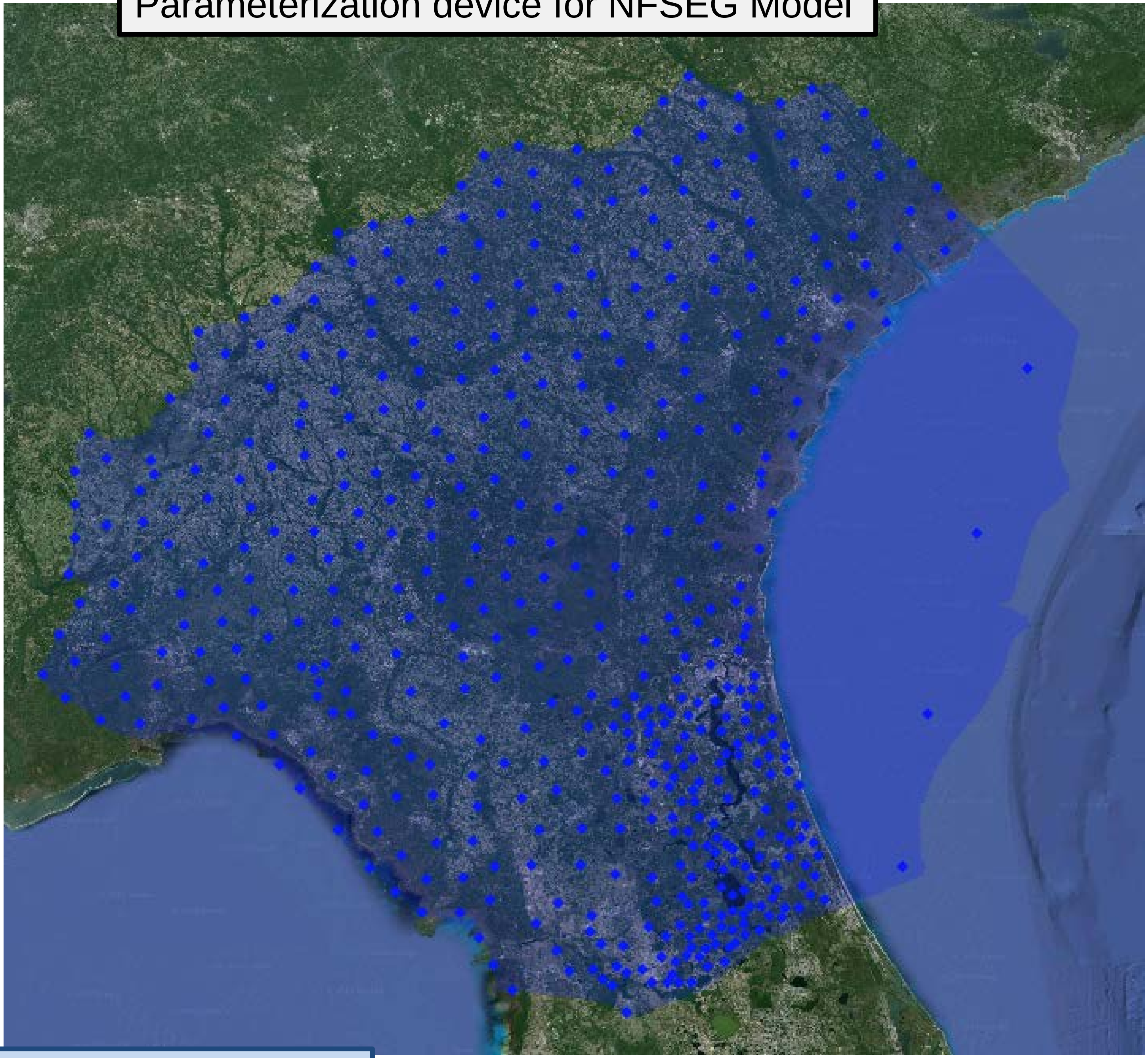




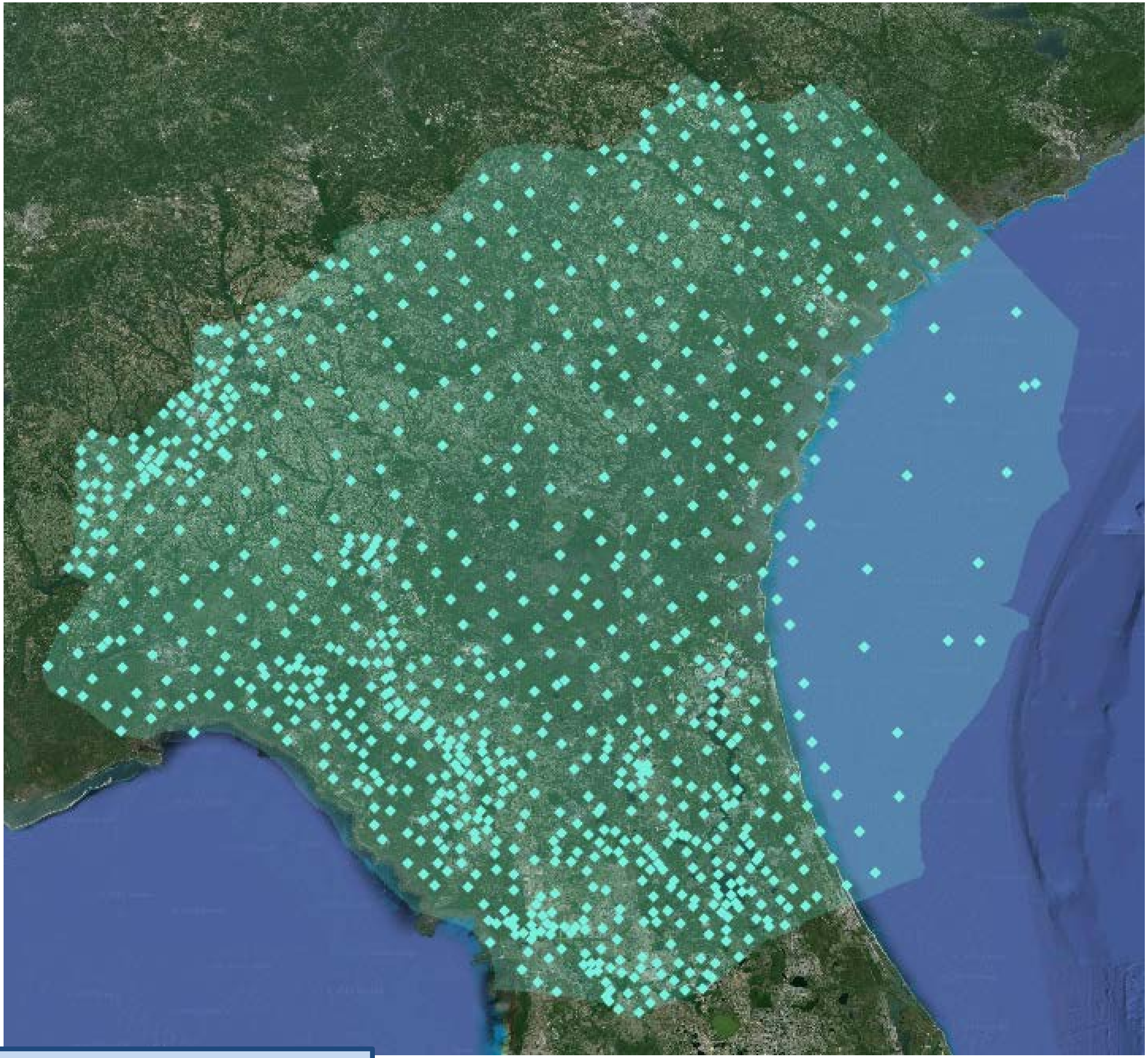


Big problem: how do you get these parameter fields to fit a calibration dataset?

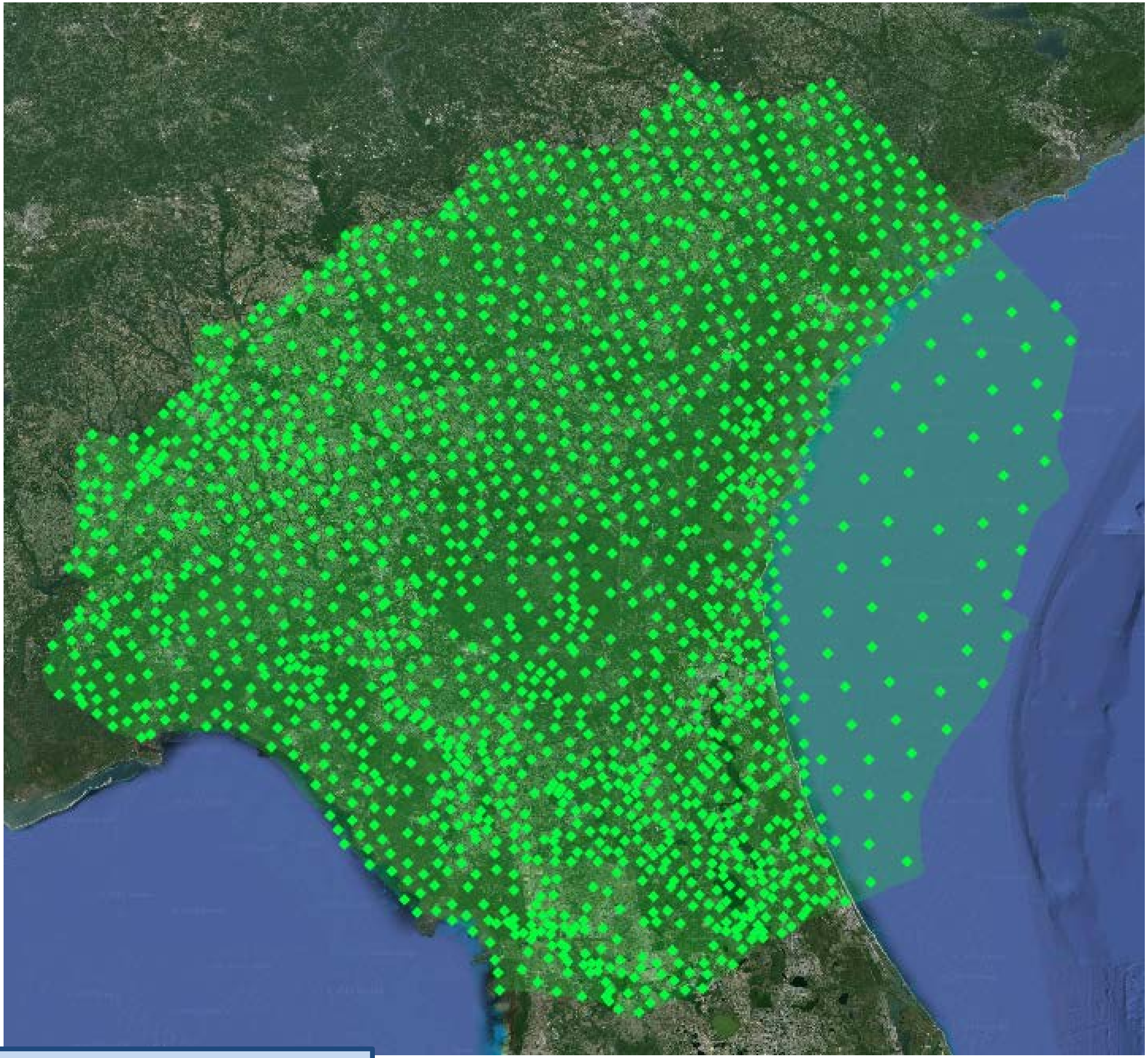
Parameterization device for NFSEG Model



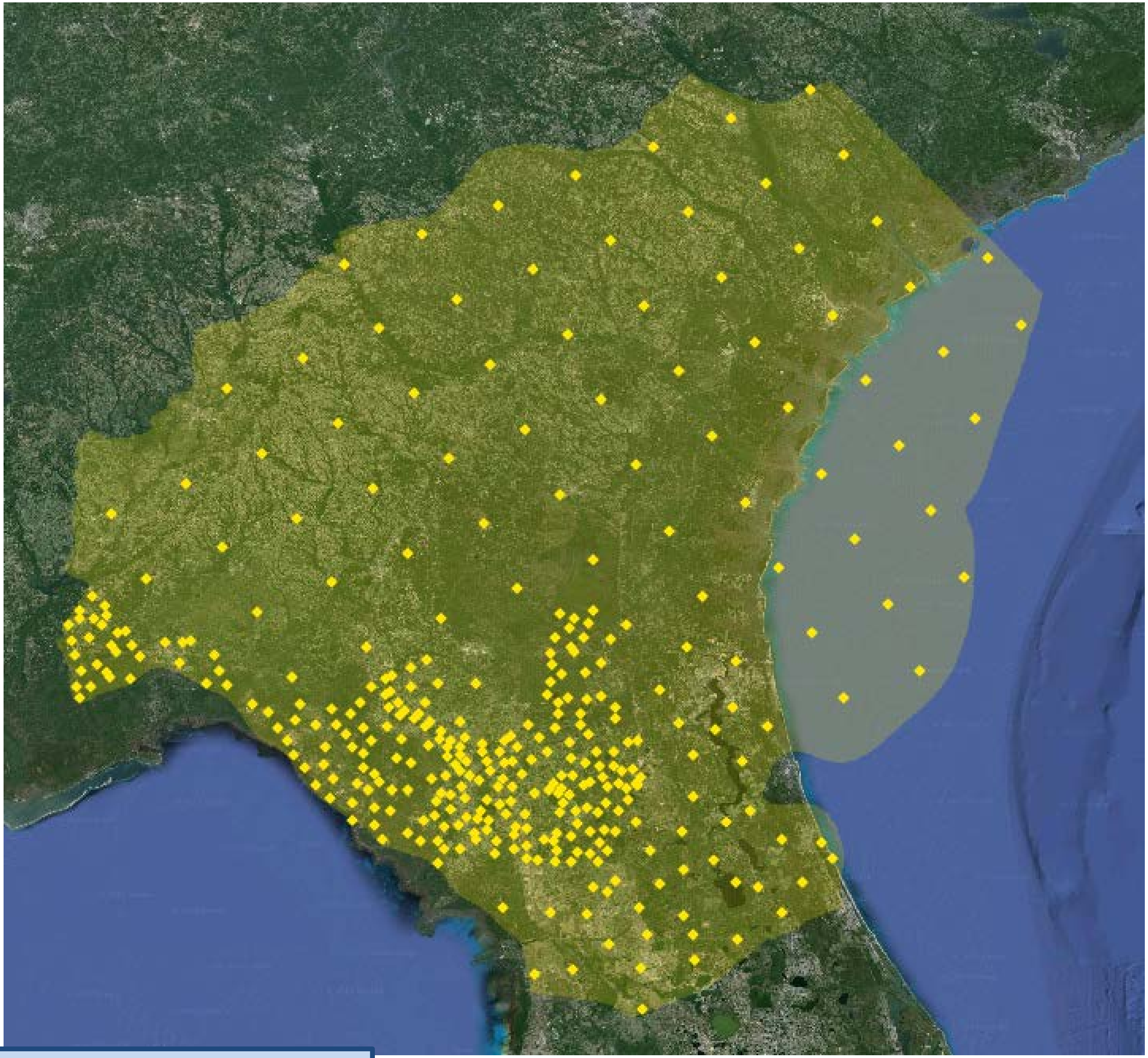
Pilot points and model domain in layer 1



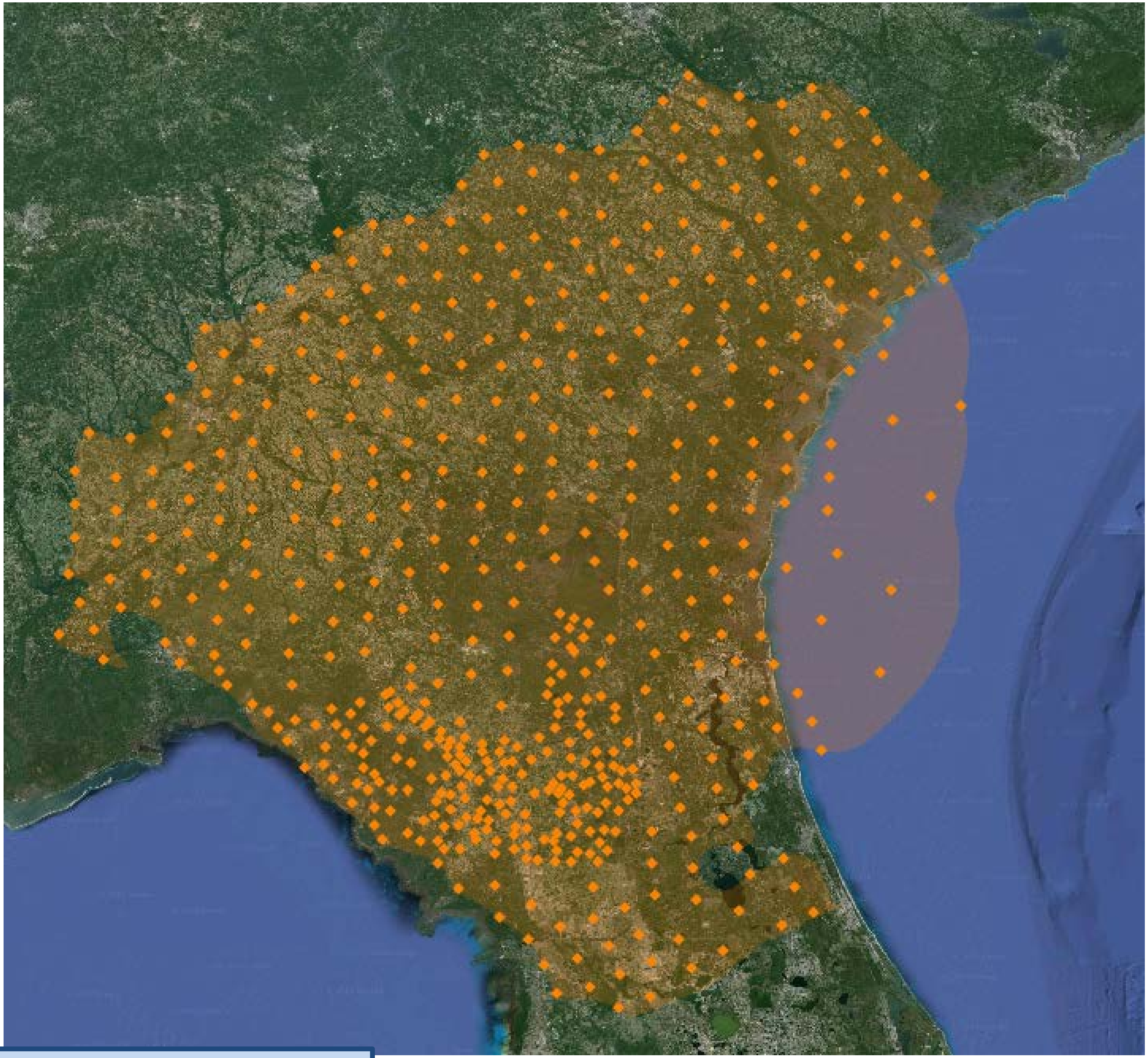
Pilot points and model domain in layer 2



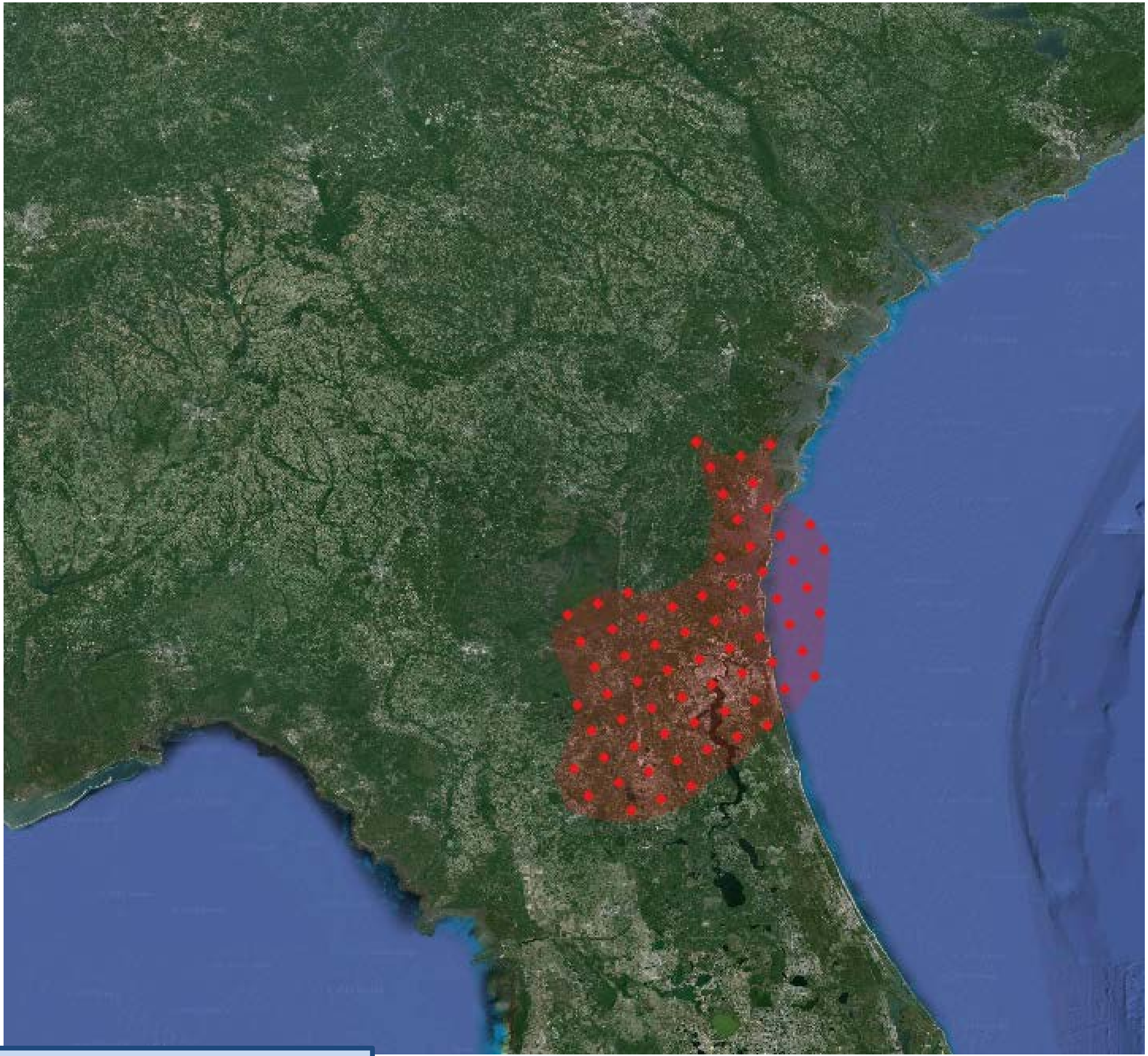
Pilot points and model domain in layer 3



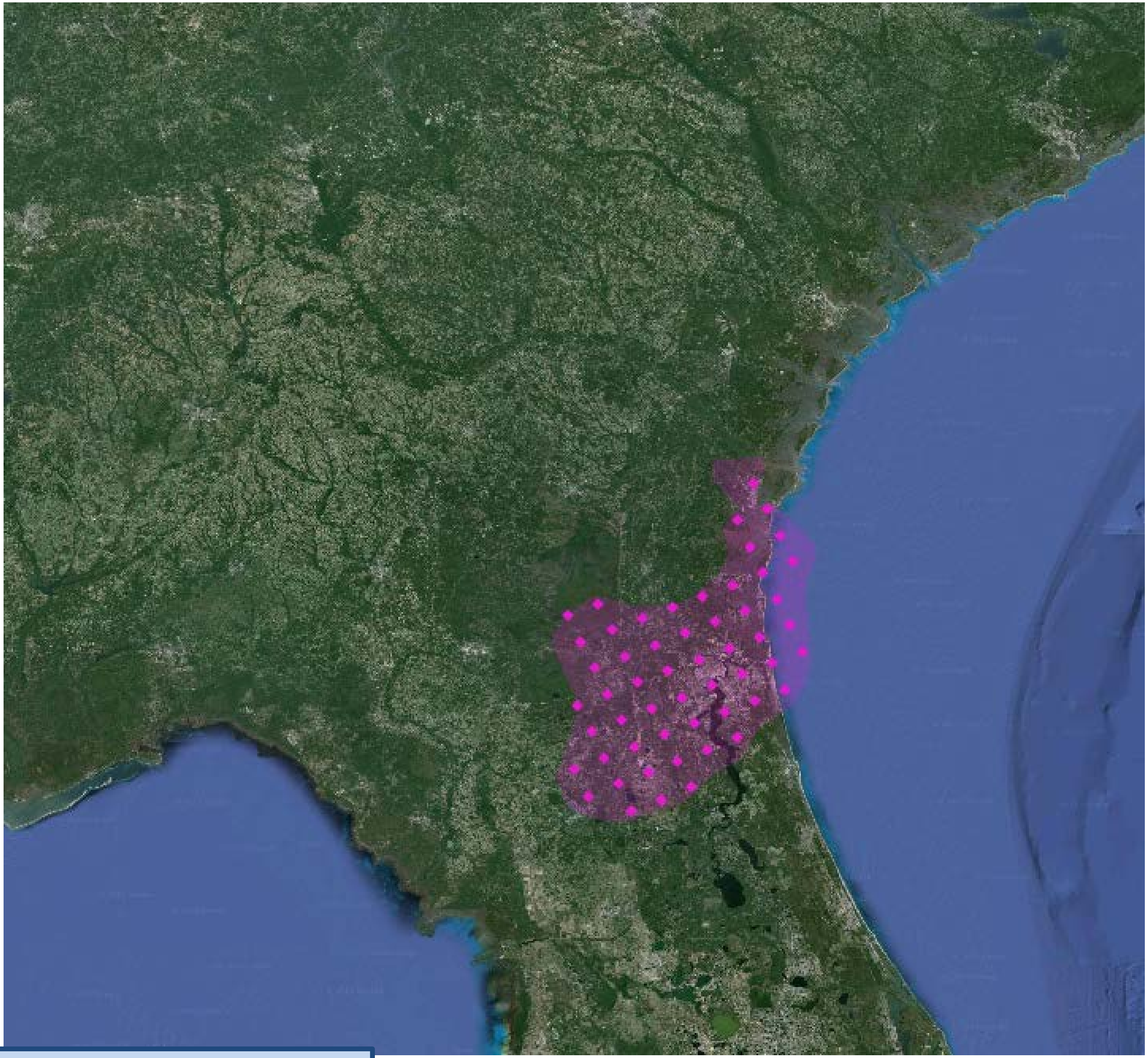
Pilot points and model domain in layer 4



Pilot points and model domain in layer 5



Pilot points and model domain in layer 6

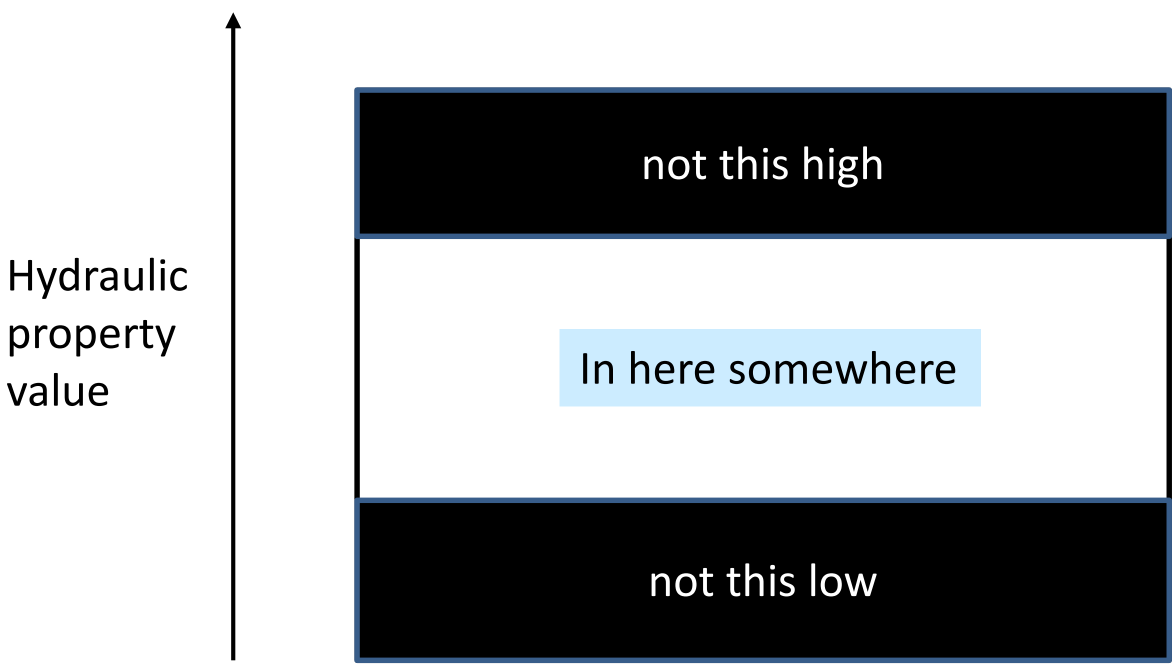


Pilot points and model domain in layer 7

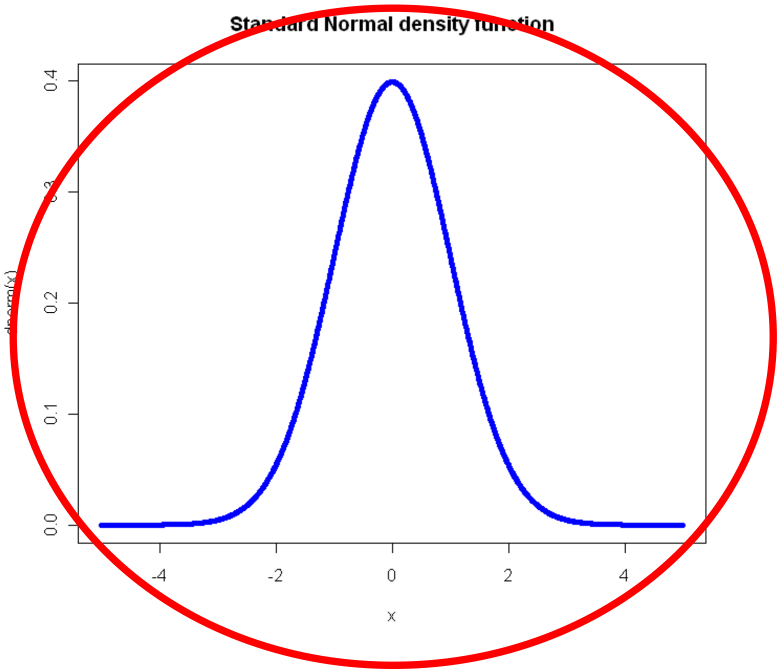
Simple Geostatistical Model

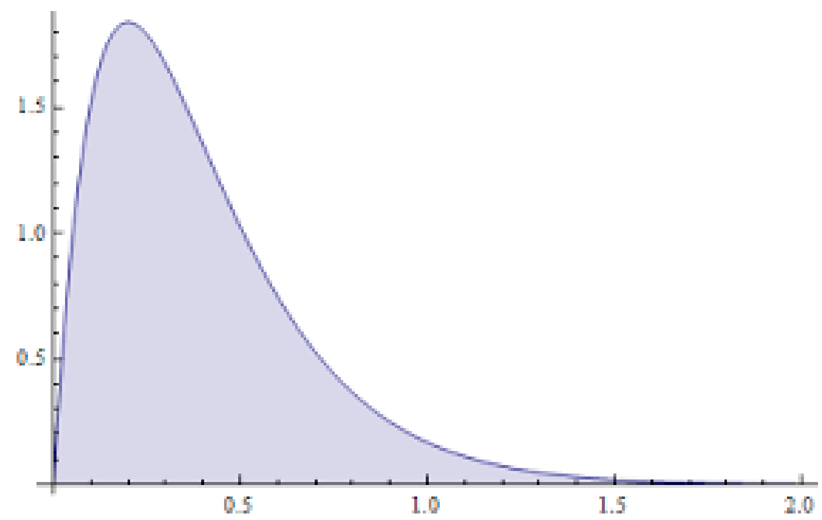
We require two things:-

- the uncertainty of each parameter
- the degree of spatial correlation between parameters



Probability Density Functions

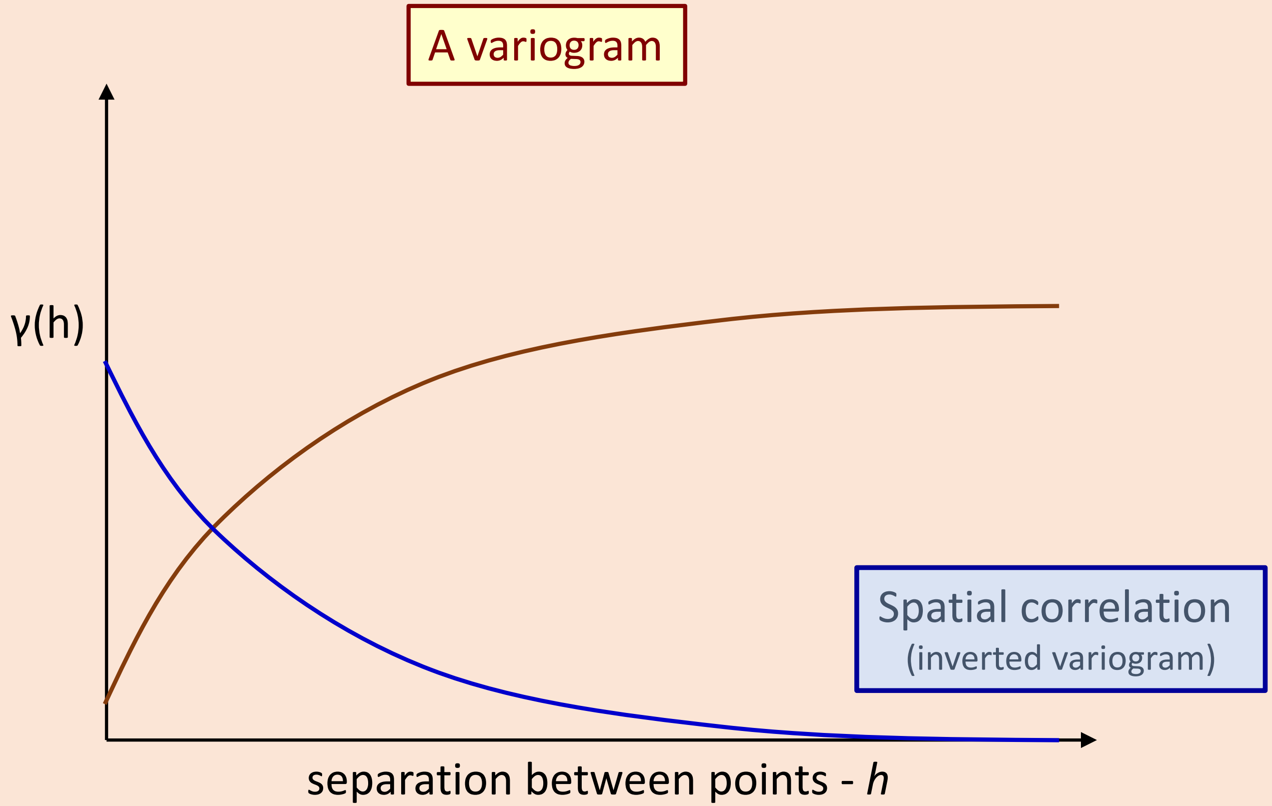




Select this. Why?

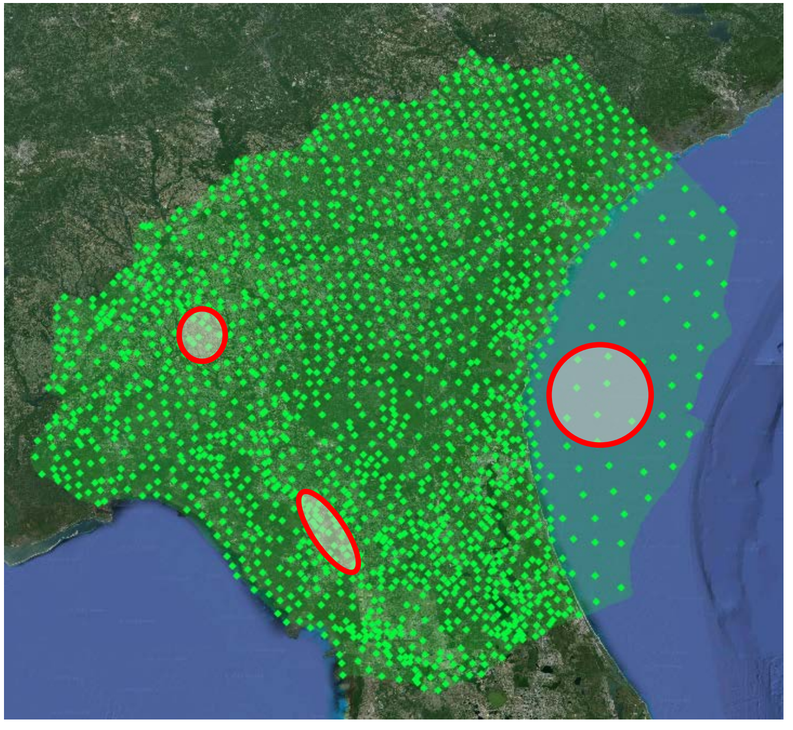
- No software/experience exists for building complex geostatistical models on a regional scale.
- Constraining random parameter fields to fit a calibration dataset is numerically difficult. Simplifying assumptions are needed.

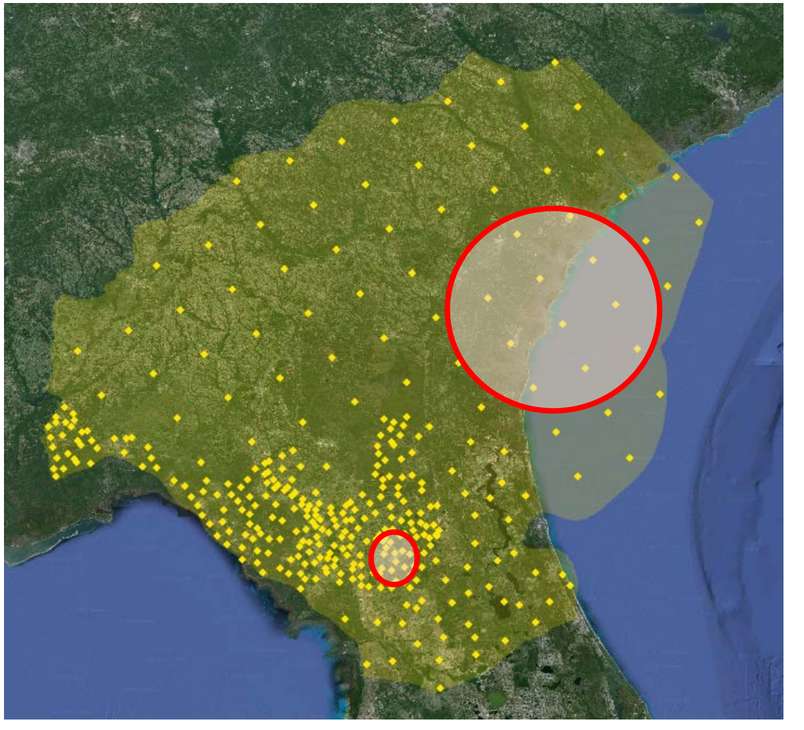
Spatial Correlation



$$\gamma(h) = E\left\{(k(x) - k(x+h))^2\right\}$$

Spatial Correlation





Pilot points are not statistically independent.

Spatial correlation depends on local pilot point spatial density.

Local pilot point density depends on local information content.

Result

$$P(k|h) \propto P(h|\,k)\, P(k)$$

Prior parameter covariance matrix

C(k)

In practice

What is possible based on expert knowledge

Enforcing Calibration Constraints on Random Parameter Fields

Step 1

Calibrate model using PEST

- Fit the calibration dataset as well as possible
- Calculate a Jacobian matrix based on best-fit parameters…

$$
\begin{bmatrix}
\partial o_1 / \partial p_1 & \partial o_1 / \partial p_2 & \partial o_1 / \partial p_3 & \partial o_1 / \partial p_4 & etc \\
\partial o_2 / \partial p_1 & \partial o_2 / \partial p_2 & \partial o_2 / \partial p_3 & \partial o_2 / \partial p_4 & \\
\partial o_3 / \partial p_1 & \partial o_3 / \partial p_2 & \partial o_3 / \partial p_3 & \partial o_3 / \partial p_4 & \\
\partial o_4 / \partial p_1 & \partial o_4 / \partial p_2 & \partial o_4 / \partial p_3 & \partial o_4 / \partial p_4 & \\
\partial o_5 / \partial p_1 & \partial o_5 / \partial p_2 & \partial o_5 / \partial p_3 & \partial o_5 / \partial p_4 & \\
\partial o_6 / \partial p_1 & \partial o_6 / \partial p_2 & \partial o_6 / \partial p_3 & \partial o_6 / \partial p_4 & \\
\partial o_7 / \partial p_1 & \partial o_7 / \partial p_2 & \partial o_7 / \partial p_3 & \partial o_7 / \partial p_4 & \\
\partial o_8 / \partial p_1 & \partial o_8 / \partial p_2 & \partial o_8 / \partial p_3 & \partial o_8 / \partial p_4 & \\
 & & & & \\
etc & & & &
\end{bmatrix}
$$

$$\mathbf{J}_{i,j} = \frac{\partial o_i}{\partial p_j}$$

Step 2

Use linearized form of Bayes equation to calculate a posterior (i.e. post-calibration) parameter covariance matrix from the prior parameter covariance matrix.

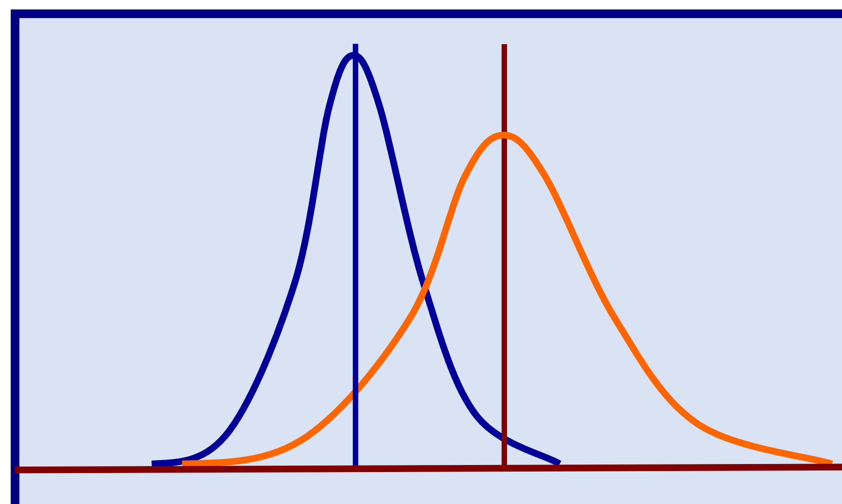
Use PREDUNC7 utility from the PEST suite.

$$C'(\mathbf{k}) = C(\mathbf{k}) - C(\mathbf{k})\mathbf{J}^t[\mathbf{J}C(\mathbf{k})\mathbf{J}^t + C(\boldsymbol{\varepsilon})]^{-1}\mathbf{J}C(\mathbf{k})$$

Prior covariance matrix

Reduced through history matching

Posterior covariance matrix

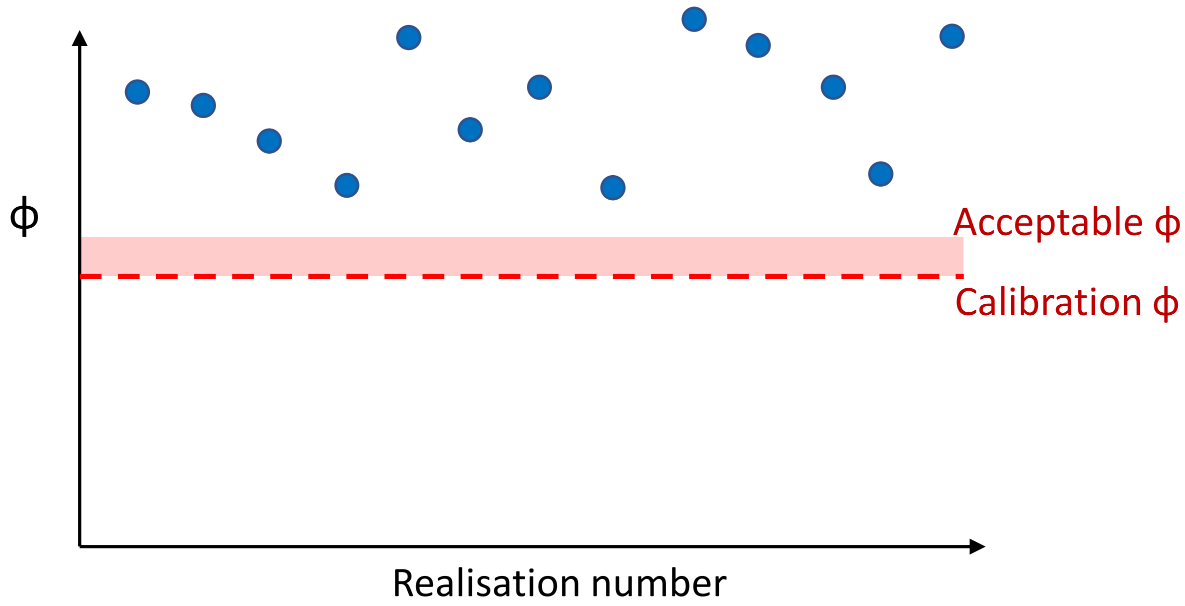


Step 3

Sample the linear approximation to the posterior parameter distribution:-

- Samples are centred on the calibrated parameter field
- Post-calibration standard deviations and spatial correlations expressed by C′(k)
- Run the model to obtain objective functions (quantify model-to-measurement misfit)

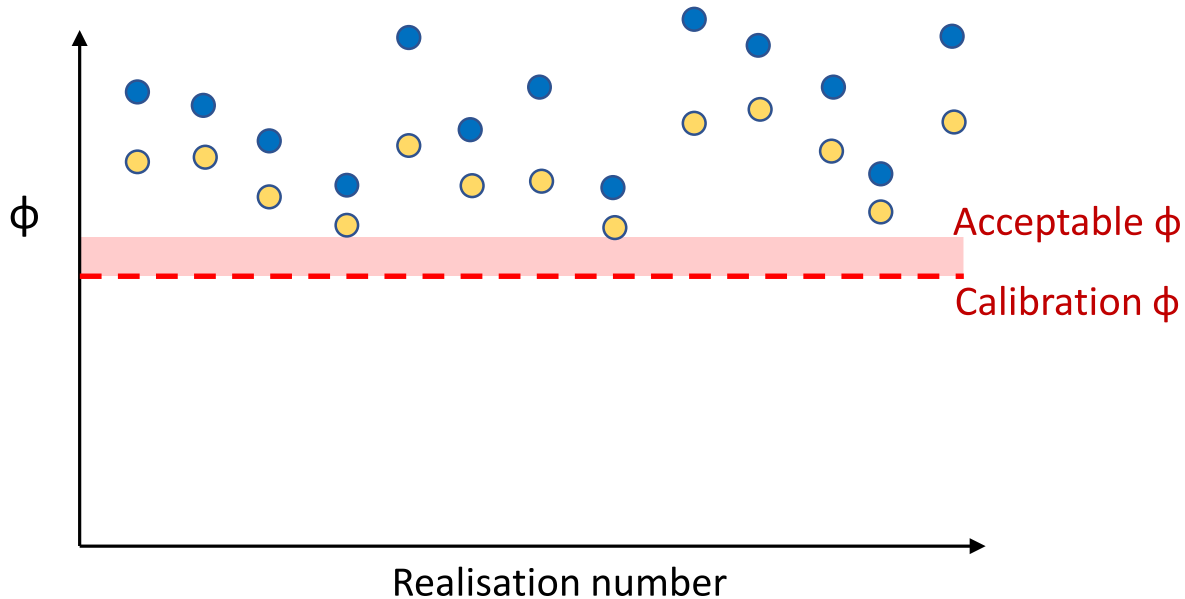
Use RANDPAR1 utility from the PEST suite.



Step 4

- undertaking singular value decomposition of the Jacobian matrix
- refine random parameter fields by extracting solution space components and replacing them with those of calibrated model.

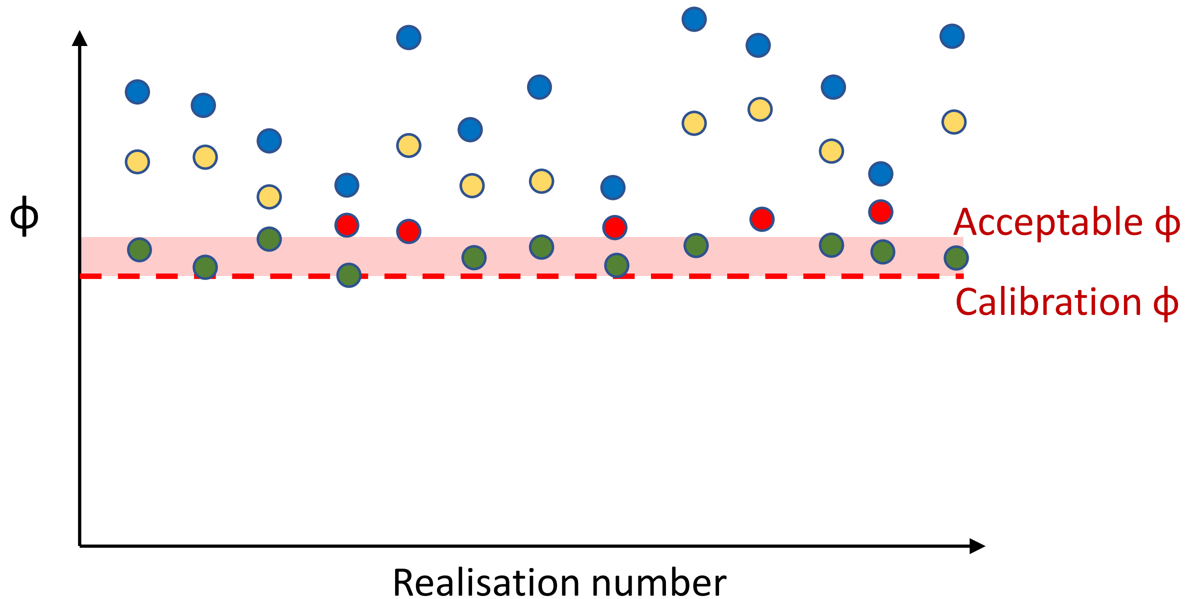
Use PNULPAR utility from the PEST suite.



Step 5

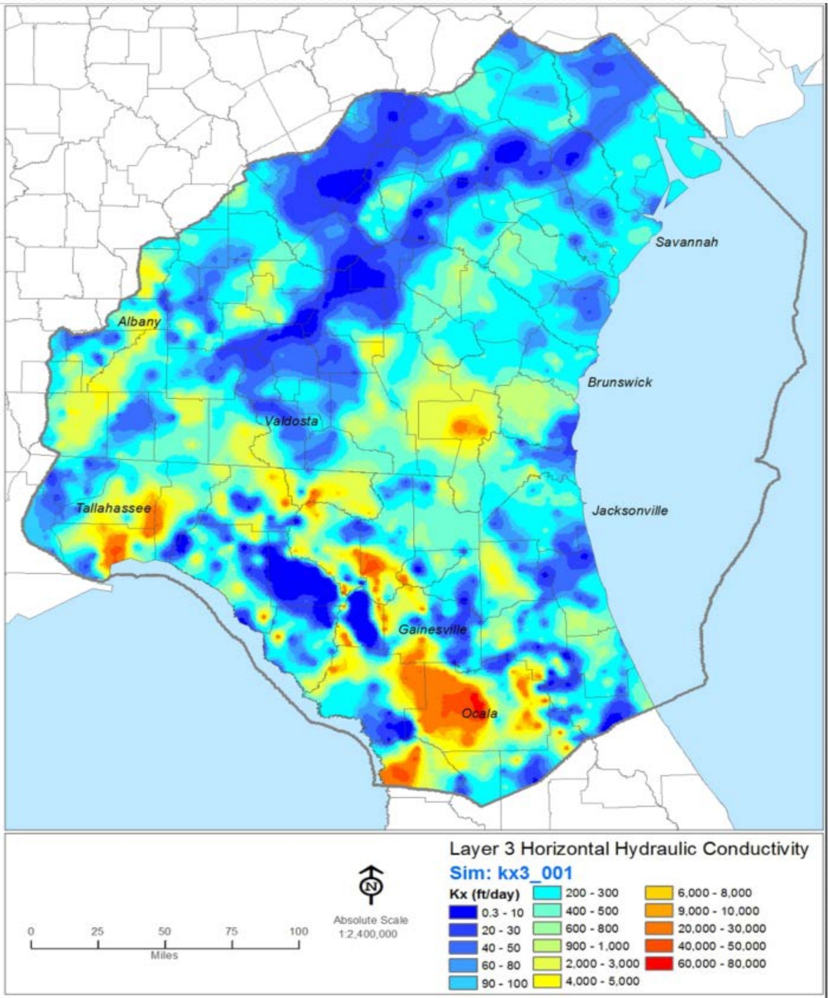
- adjust parameter fields using a single PEST iteration
- adjustment is numerically cheap; use same Jacobian matrix

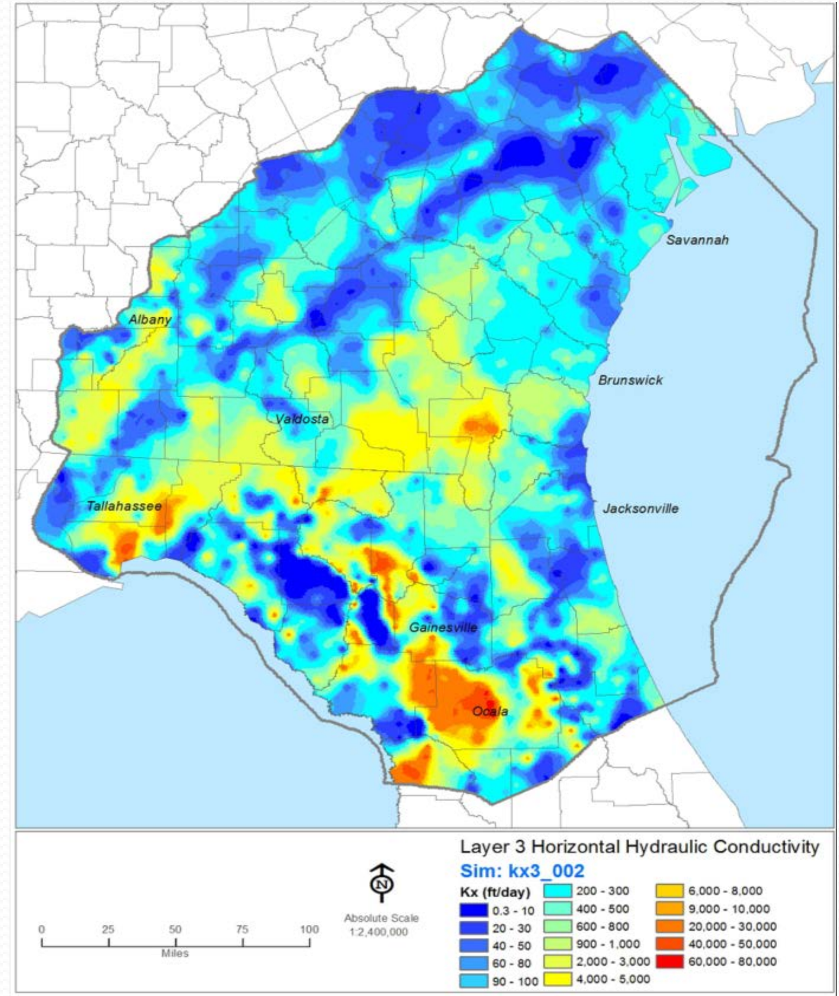
Run PEST with the “/i” switch



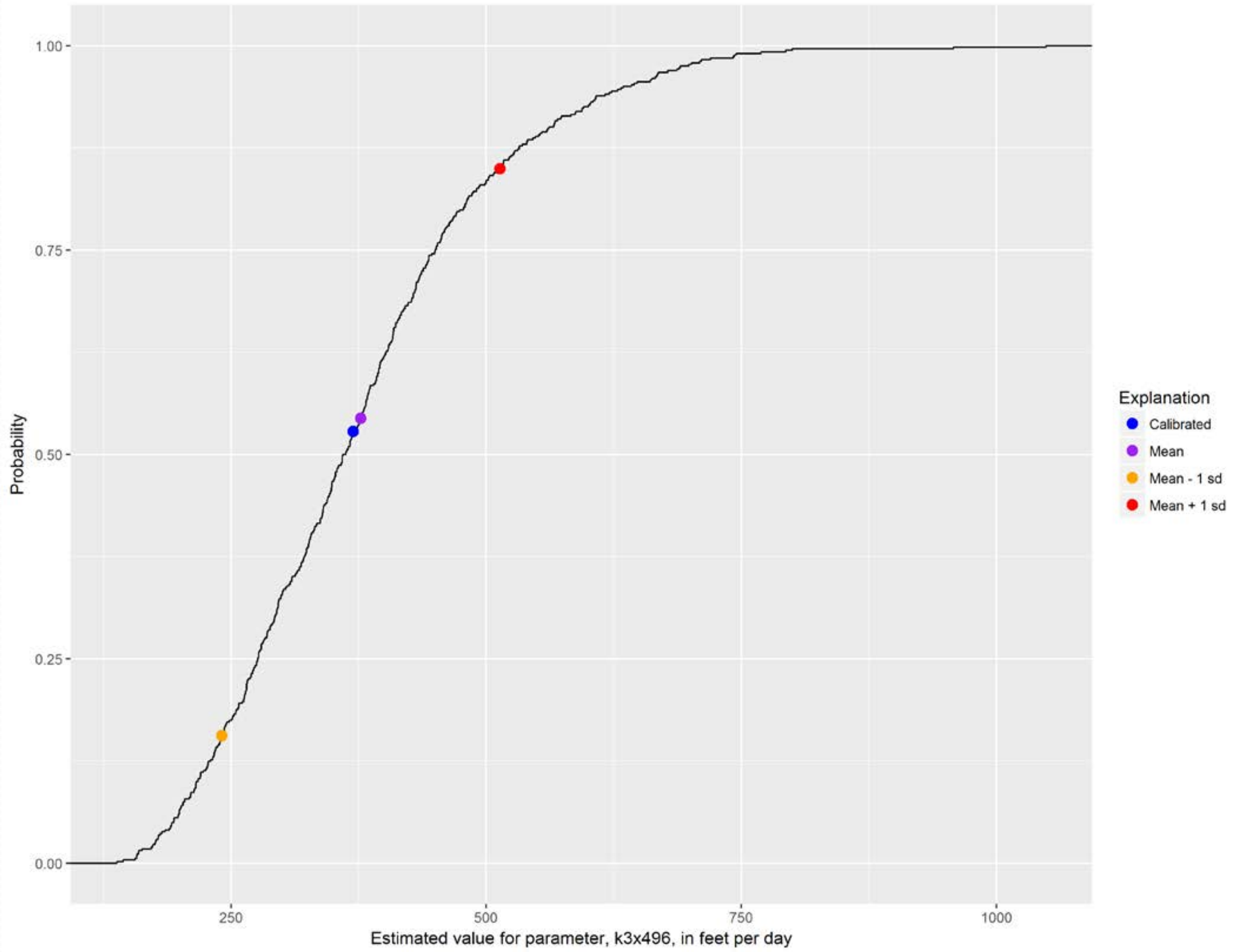
Outcomes

Examples of Randomly Generated Parameters

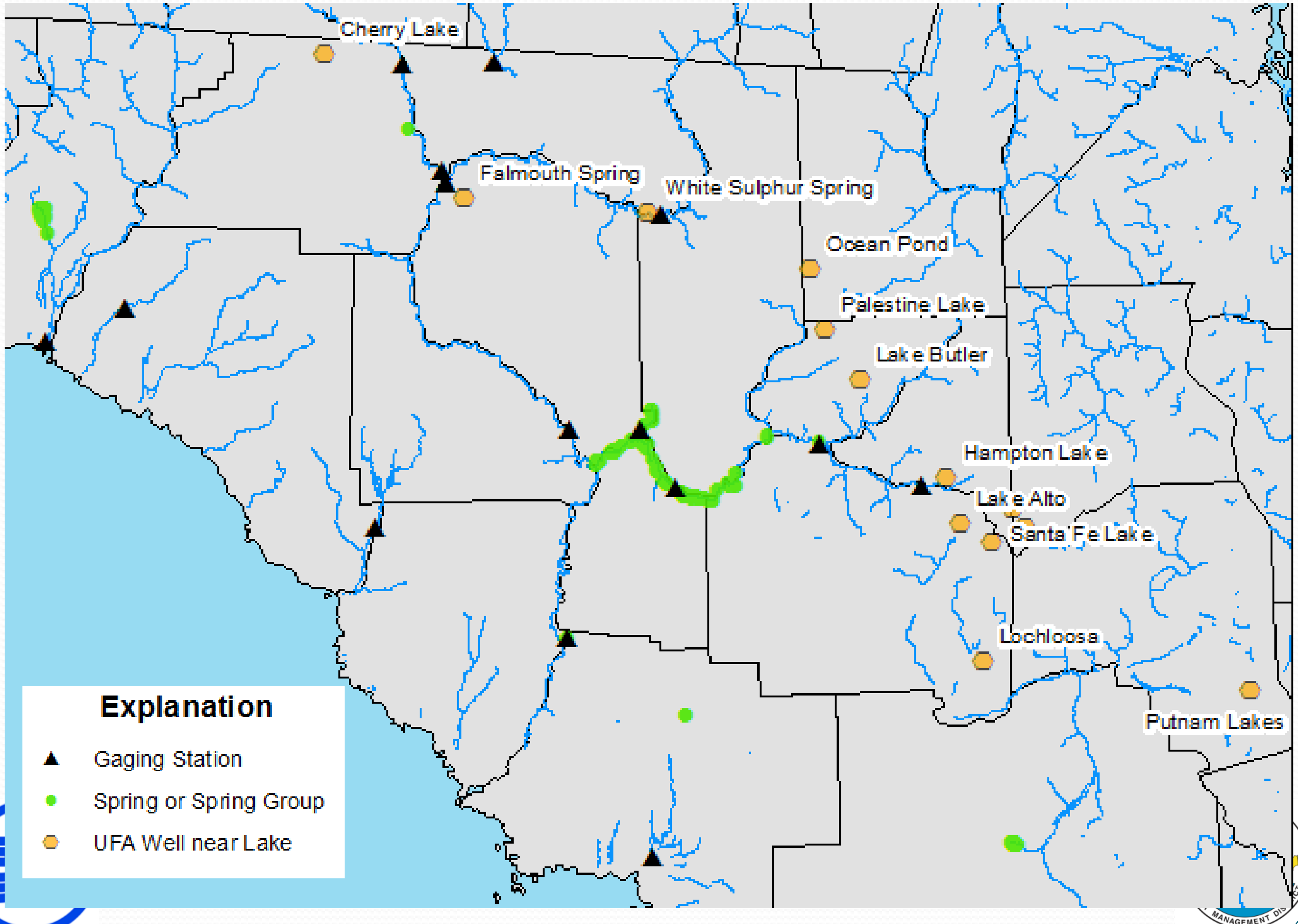




Parameter Uncertainty: Estimated Probability Distributions



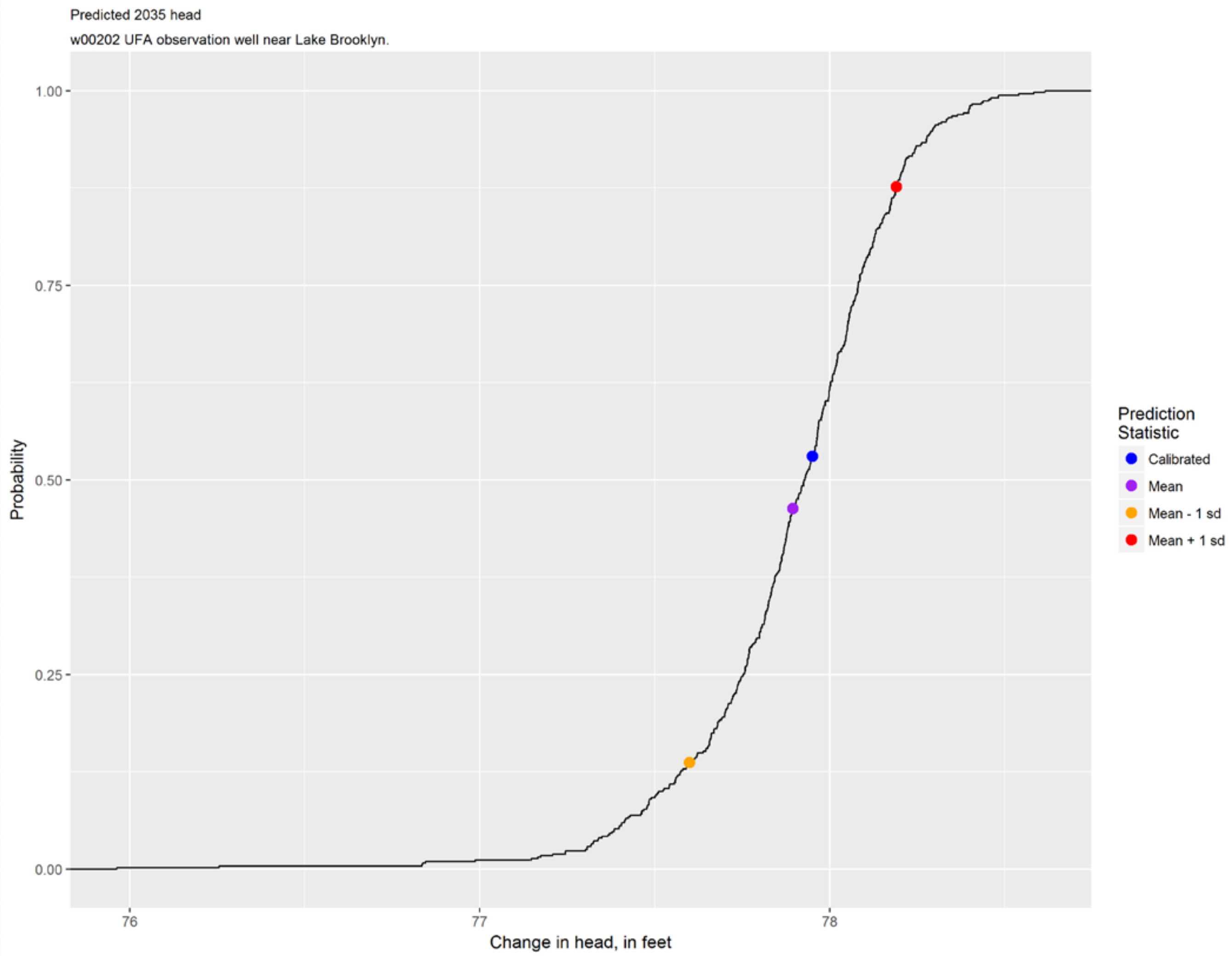
Prediction Locations



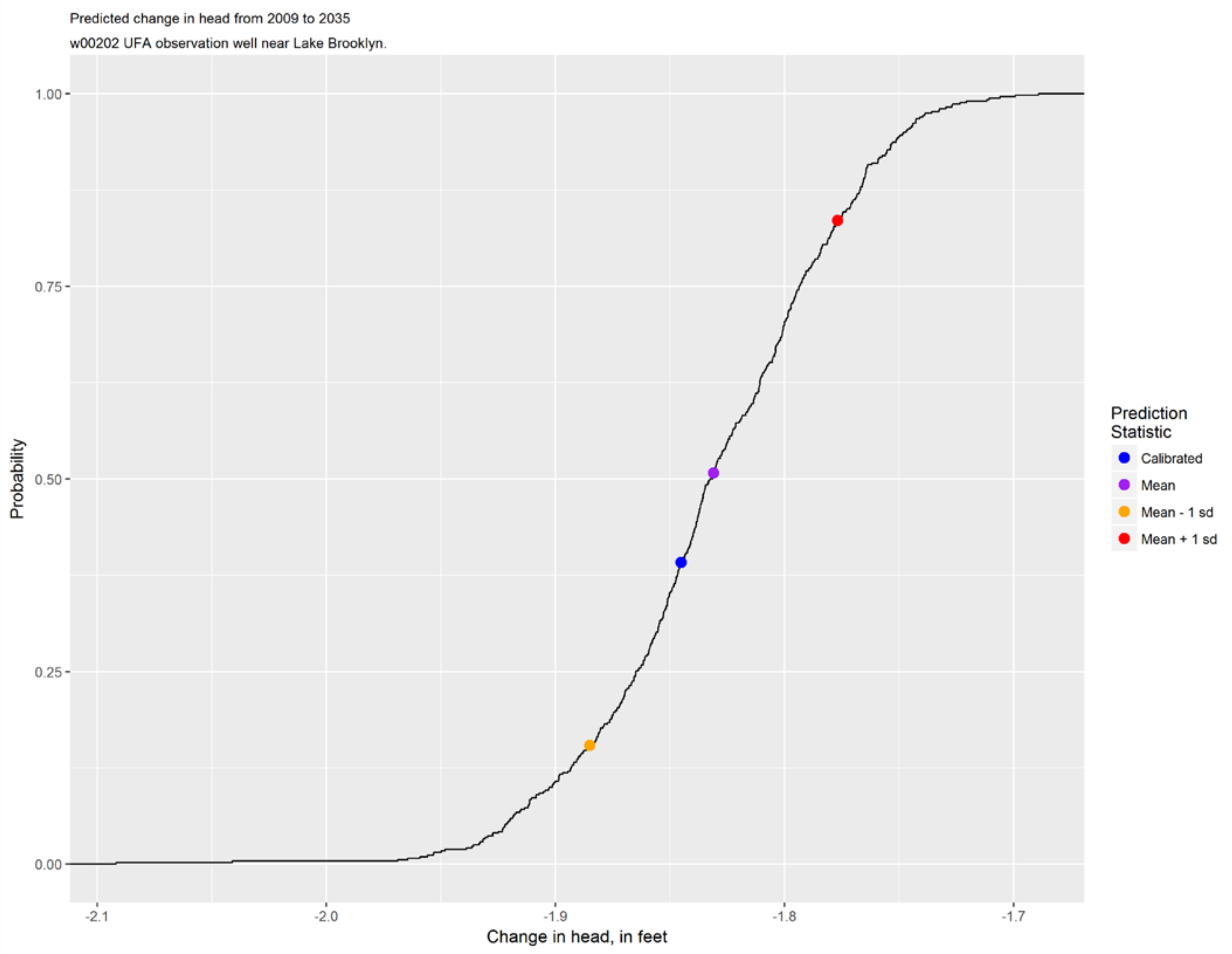
Predictive Uncertainty

Prediction Location and Type	Prediction Units	Mean of 2035 Predicted Value	Standard Deviation of 2035 Predicted Value	Mean of Predicted Change from 2009 to 2035	Standard Deviation of Predicted Change from 2009 to 2035
UFA observation well near Lake Brooklyn	Feet	77.9	0.3	-1.8	0.1
UFA observation well near Lake Geneva	Feet	77.7	0.3	-1.9	0.1
Ichetucknee River at US HWY27 near Hildreth	Flow	-269.	4.8	7.5	0.3
Santa Fe River near Fort White	Flow	-707.	6.6	15.4	0.8

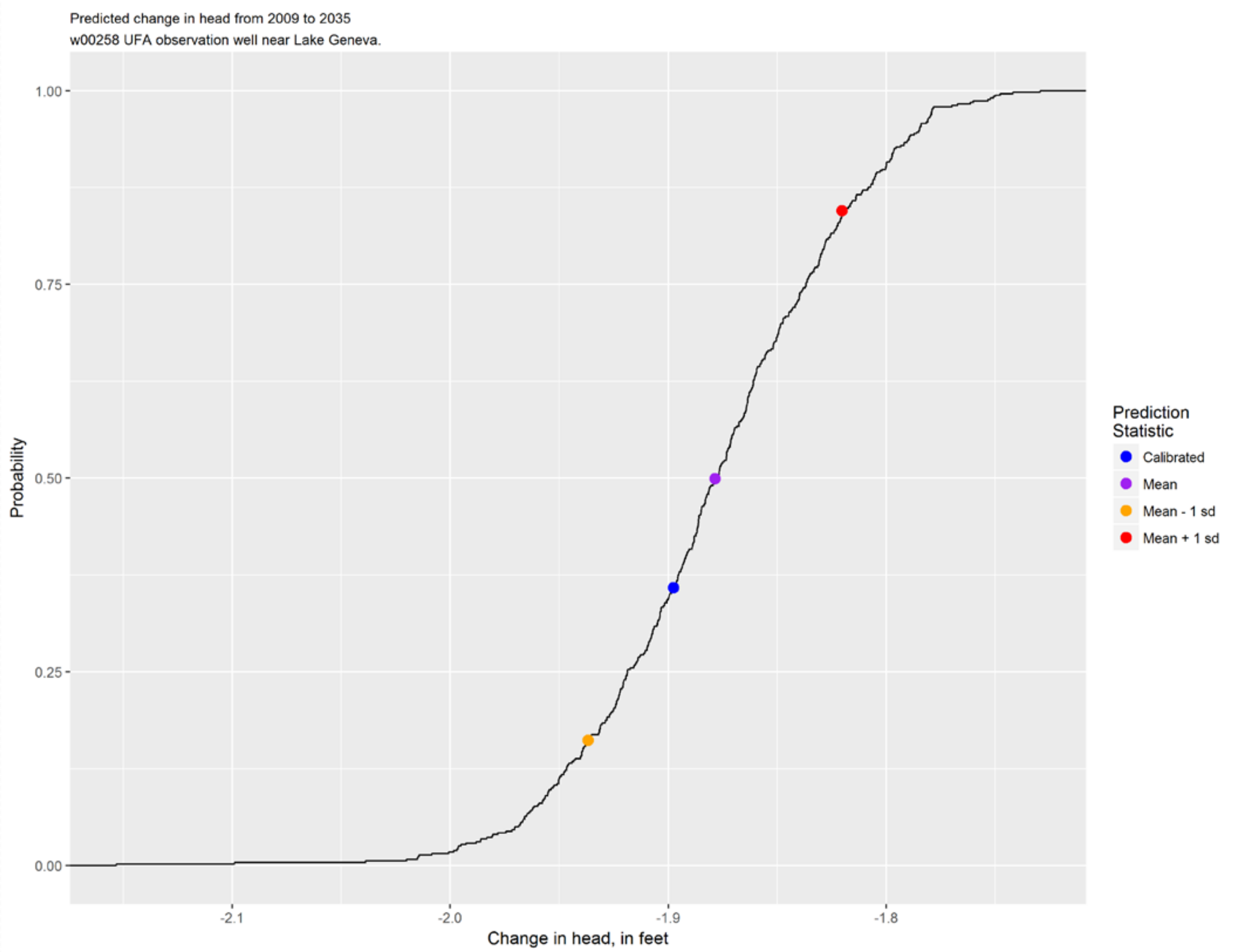
Predictive Uncertainty: Estimated Probability Distributions



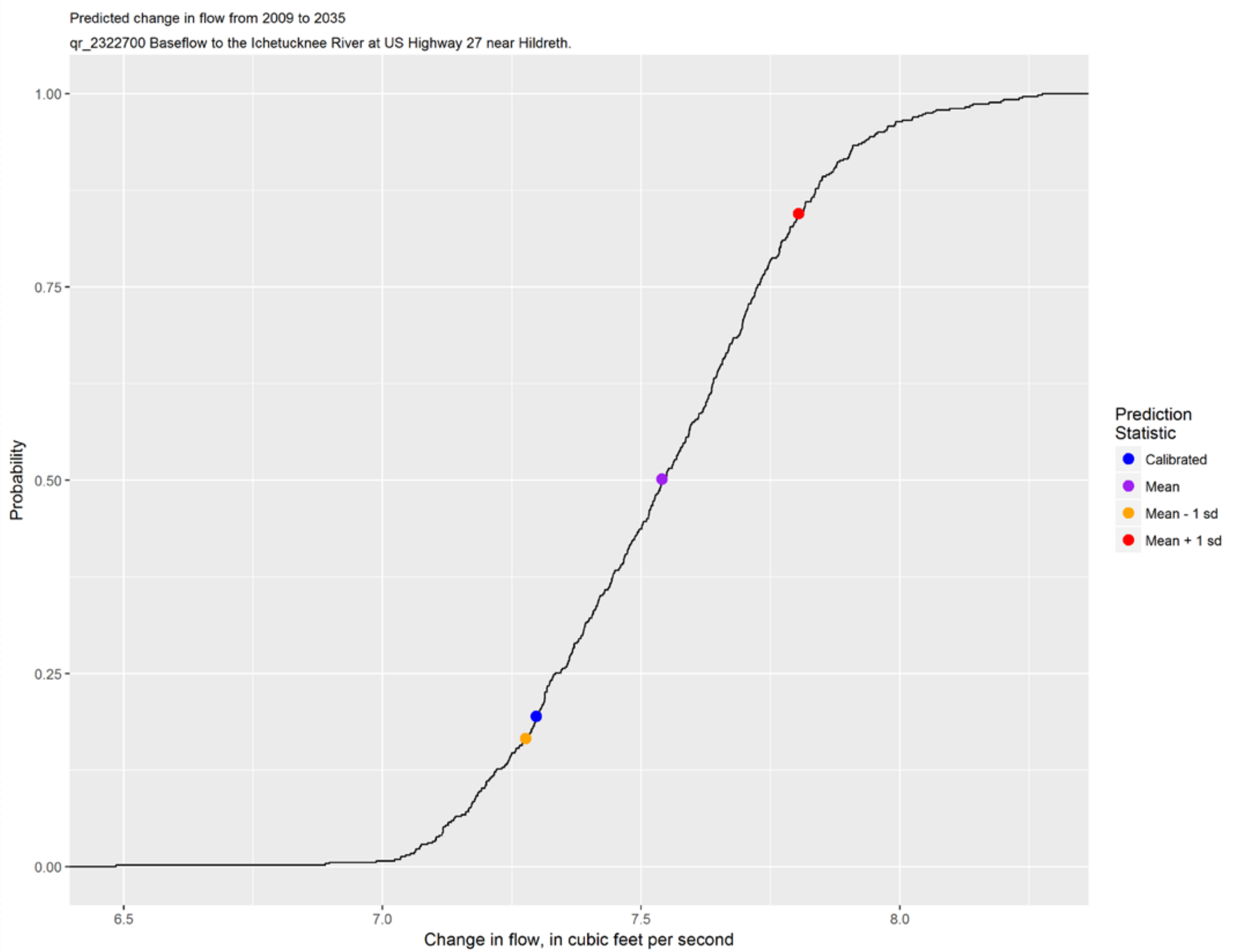
Predictive Uncertainty: Estimated Probability Distributions



Predictive Uncertainty: Estimated Probability Distributions



Predictive Uncertainty: Estimated Probability Distributions



Predictive Uncertainty: Estimated Probability Distributions